

NAME AND ID NUMBER:

Problem	1	2	3	4	5	6	Total
Score							

MAT 319 Final

May 19, 2020

Total maximal score = 60.

ALL THE WORK YOU WILL DO IN THIS EXAM SHOULD BE YOUR OWN, WITHOUT CONSULTING OTHER PEOPLE OR ONLINE SOURCES OF ANY KIND DURING THIS EXAM. SHOW ALL YOUR WORK. WRITE PROOFS CAREFULLY, CLEARLY AND COMPLETELY.

- (1) (10 points) Prove that

$$1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

for all $n \in \mathbb{N}$.

- (2) (10 points) Prove that the equation $x^2 = 5$ does not have a rational solution.

- (3) (10 points) Show that $\lim(\sqrt{n+1} - \sqrt{n}) = 0$.

- (4) (10 points) Determine and prove if the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$$

- (5) (10 points) Use the Mean Value Theorem to prove that

$$|\cos x - \cos y| \leq |x - y|$$

for all x, y in \mathbb{R} .

- (6) (10 points) If f is Riemann integrable on $[a, b]$ and $|f(x)| \leq M$ for all $x \in [a, b]$, show that

$$\left| \int_a^b f(x) dx \right| \leq M(b-a).$$

End of Examination.