ERRATUM TO “DEMAILLY’S NOTION OF ALGEBRAIC HYPERBOLICITY” AND “BOUNDEDNESS IN FAMILIES WITH APPLICATIONS TO ARITHMETIC HYPERBOLICITY”

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The proofs of Theorem 1.14 in [1] and of its “pseudofied” extension, Theorem 1.26 in [2], contain a gap in the argument. This mistake does not affect any of the other results nor proofs in the rest of the papers.

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Let X be a bounded projective variety over an algebraically closed field k of characteristic zero. By definition, for every ample line bundle L and smooth projective curve C and every nonconstant morphism $f\colon C \to X$, the boundedness of X implies the existence of a bound on $\deg f^*L$ depending only on X, L, and C.

Theorem 1.14 of [1] asserts that one can bound $\deg f^*L$ uniformly in the genus of C. More precisely, given a projective bounded variety X, for every integer g and every ample line bundle L on X, there exists a real number $\alpha(X, L, g)$ such that, for every smooth projective curve C of genus g and every non-constant morphism $f\colon C \to X$, one has

$$\deg f^*L \leq \alpha(X, L, g).$$

Let us explain the mistake in the proof of Theorem 1.14 of [1] (the same issue occurs in the proof of Theorem 1.26 of [2]). In the notation of that proof, one first shows that the Hom-scheme $\mathrm{Hom}_k(Y, X \times Z)$ is of finite type over k. Our proof then asserts, without justification, that the Hom-scheme $\mathrm{Hom}_Z(Y, X \times Z)$ is also of finite type. This statement is likely true but requires a proof.

Consider the natural morphism of Hom-schemes

$$\mathrm{comp}\colon \mathrm{Hom}_k(Y, X \times Z) \to \mathrm{Hom}_k(Y, Z), \qquad f \mapsto \pi_Z \circ f,$$

where $\pi_Z\colon X \times Z \to Z$ is the projection. In the original argument we incorrectly identified $\mathrm{Hom}_Z(Y, X \times Z)$ with the scheme-theoretic fibre of comp over the structure morphism $Y \to Z$ (viewed as a k-point of $\mathrm{Hom}_k(Y, Z)$). Although this fibre is closed in $\mathrm{Hom}_k(Y, X \times Z)$, and therefore of finite type, it cannot be identified with $\mathrm{Hom}_Z(Y, X \times Z)$ in any natural sense. (Rather, it should be viewed as the Weil restriction of $\mathrm{Hom}_Z(Y, X \times Z)$ from Z to $\mathrm{Spec}\, k$.) We do not know how to repair this proof.

Theorem 1.14 in [1] (resp. Theorem 1.26 in [2]) is thus a uniform boundedness statement which, in light of this gap, remains unproven. We note, however, that such a statement would follow from Lang’s conjecture that every bounded projective variety is in fact *algebraically hyperbolic*.

References

[1] A. Javanpeykar and L. Kamenova. Demailly’s notion of algebraic hyperbolicity: geometricity, boundedness, moduli of maps. *Math. Z.*, 296(3-4):1645–1672, 2020.

[2] R. van Bommel, A. Javanpeykar, and L. Kamenova. Boundedness in families with applications to arithmetic hyperbolicity. *J. Lond. Math. Soc. (2)*, 109(1):Paper No. e12847, 51, 2024.