MAT544 HW4

Due 10/18 in class. Each problem is worth 10 points

Problem 1. Find a ring A and a multiplicative set $S \subset A$ such that the relation $(a, s) \sim (b, t) \Leftrightarrow at = bs$ is not an equivalence relation. [Hint: S must contain zero divisors.]

Problem 2. Let A be a ring and $S \subset A$ a multiplicative set. Assume $S^{-1}A \neq 0$. Prove that the ring $S^{-1}A$ and the homomorphism $A \to S^{-1}A$ are uniquely determined (up to isomorphism) by the universal property: if $f : A \to B$ is a ring homomorphism and $f(S)$ consists of units, then there exists a unique ring homomorphism $f' : S^{-1}A \to B$ with the property that f is the composite $A \to S^{-1}A \to B$.

Problem 3. Let $S \subset T$ be multiplicative sets of a ring A. Write $A' = S^{-1}A$ and $\phi : A \to A'$ for the ring of fractions with respect to S, and $T' = \phi(T)$. Prove that $T^{-1}A = T'^{-1}A'$. In other words, a composite of two localisations is a localisation.

Problem 4. Let A be a ring and $P \subset Q$ are prime ideals. Then A_P is a localisation of A_Q at the prime ideal PA_Q.