MAT544 HW1

Due 9/6 in class. Each problem is worth 10 points

Problem 1. Let $\phi : A \to B$ be a ring homomorphism. Prove that ϕ^{-1} takes prime ideals of B to prime ideals of A. In particular, if $A \subset B$ and P is a prime ideal of B, then $A \cap P$ is a prime ideal of A.

Problem 2. Prove or give a counterexample:

(a) the intersection of two prime ideals is prime;

(b) the ideal $P_1 + P_2$ generated by two prime ideals P_1 and P_2 is again prime;

(c) if $\phi : A \to B$ is a ring homomorphism, then ϕ^{-1} takes maximal ideals of B to maximal ideals of A.

Problem 3. Let k be an algebraically closed field. Prove that the prime ideals of $k[X, Y]$ are the following: 0; (f) for an irreducible $f \in k[X, Y]$; and maximal ideals m. Moreover, each maximal ideal is of the form $m = (X - a, X - b)$ for some $a, b \in k$.

Problem 4. Prove that the prime ideals of $\mathbb{Z}[Y]$ are the following: 0; (f) for an irreducible $f \in \mathbb{Z}[Y]$; and maximal ideals m. Moreover, each maximal ideal is of the form $m = (p, g)$, where p is a prime number and $g \in \mathbb{Z}[Y]$ is a polynomial whose reduction modulo p is irreducible in $\mathbb{F}_p[Y]$.