MAT402 HW1

These are practice problems.

Problem 1. Using Zorn's lemma, show that any *atlas*, i.e., a collection of charts on a manifold M, is contained in a unique maximal atlas.

Problem 2. Let $Gr(k, \mathbb{R}^n)$ be the Grassmannian of k-planes through the origin in $\mathbb{R}^n$. Show that $Gr(k, \mathbb{R}^n)$ has an atlas with $\binom{n}{k}$ charts each homeomorphic to $\mathbb{R}^{k(n-k)}$.

Problem 3. Let $Gr(k, \mathbb{C}^n)$ be the Grassmannian of k-planes through the origin in $\mathbb{C}^n$. Show that $Gr(k, \mathbb{C}^n)$ has an atlas with $\binom{n}{k}$ charts each homeomorphic to $\mathbb{C}^{k(n-k)}$. This gives $Gr(k, \mathbb{C}^n)$ the structure of a complex manifold.