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Introduction to Linear Algebra
MAT 211, Spring 2009

Exercises for Midterm I

1. Solve the system and verify (check) your solution.

$$\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 1 \\ -2x_1 + x_2 + 2x_3 - x_4 = 2 \\ 4x_1 + 3x_2 - 4x_3 + 3x_4 = 0. \end{cases}$$

2. For each value of a constant a, find the solution of the system

$$\begin{cases} -ax + y + 2z = 3 \\ 2x + (a+2)y + z = 2 \\ (1-a)x + y + z = 2. \end{cases}$$

3. Find the matrix (with respect to the standard basis) of the reflection about the line $x+2y=0$ in $\mathbb{R}^2$.

4. Find the matrix (with respect to the standard basis) of the projection onto the plane $x+y-z=0$ in $\mathbb{R}^3$.

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the clockwise rotation by $30°$ followed by the projection onto the line $2x - 3y = 0$. Is T invertible? Find the matrix of T (with respect to the standard basis).

6. For which values of a constant k is the matrix

$$A = \begin{pmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{pmatrix}$$

invertible? Find the inverse.

7. Show that the vectors $(2,0,1,3)$, $(0,0,2,2)$, $(4,1,0,1)$ and $(6,1,1,4)$ are linearly dependent. Express vector $(2,0,1,3)$ as a linear combination of the other vectors. Can vector $(0,0,2,2)$, be presented as a linear combination of the other vectors?

8. Let V be a subspace of $\mathbb{R}^4$ spanned by vectors $v_1 = (1,0,1,-1)$, $v_2 = (2,1,1,1)$, and $v_3 = (1,-1,2,1)$.
a) Does the vector $v = (0,-2,2,-11)$ belong to V?
b) For which values of a constant a does the vector $u = (2-a, 1-2a, 2, 1)$ belong to V?

9. Let V be a subspace of $\mathbb{R}^4$ given by

$$V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \,|\, x_1 - x_3 + 2x_4 = 0 \text{ and } x_1 - x_2 + x_3 - x_4 = 0\}.$$

Find a basis in V.

10. A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by the following: $T(1,1,0) = (2,1,0)$, $T(0,1,0) = (-1,2,1)$, $T(0,1,1) = (2,1,5)$. Find the matrix of T (with respect to the standard basis).

11. A linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ is defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2 - x_4,\ x_2 + x_3 + x_4,\ x_1 - x_3 - 2x_4).$$

a) Find the matrix of T with respect to the standard basis.
b) Find a basis in the kernel of T and a basis in the image of T.
c) Find the dimensions of the kernel and the image.
d) Find the rank of T.
e) Verify the Kernel -Image (Rank-Nullity) theorem for T.