MAT131 Fall 2022 Paper HW 11

Due the week of November 28 – December 2. For all problem sets, students are allowed to work together. However, the final answer you turn in must be based on your own understanding and must be in your words. Per university policy, all instances of suspected academic dishonesty will be referred to the academic judiciary.

Problem 1. Let p be a positive real number, let $f(x) = x^{p-1}$ on the interval $(0, \infty)$, let $a = 1$, and let $b > 1$ be a real number, say $b = e^B$ for a real number $B > 0$. For the Riemann integral,

$$\int_1^b x^{p-1} dx = \int_1^b x^p \frac{dx}{x},$$

what do you get if you perform an integration by direct substitution with $u = \ln(x)$, $x = e^u$ on the interval $0 < u < \infty$? For the new integrand, $g(u)$, find an antiderivative $G(u)$ of $g(u)$. Finally, simplify the back-substituted expression $G(\ln(x))$, and check that this agrees with the usual formula for this integral.

Problem 2. By the Fundamental Theorem of Calculus, an antiderivative $G(\theta)$ of $\sec(\theta)$ exists on the interval $0 < \theta < \pi/2$. Define $H(x) = G(\arccos(x))$ on the interval $0 < x < 1$. Compute $H'(x)$ using the Chain Rule to get an expression $k(x)$. Finally, antidifferentiate $k(x)$ using integration by direct substitution with $y = \sqrt{1 - x^2}$, $x = \sqrt{1 - y^2}$, on the interval $0 < y < 1$, together with the following identity,

$$\frac{2}{y^2 - 1} = \frac{1}{y - 1} - \frac{1}{y + 1}.$$

Back-substitute to find a formula for $G(\theta)$ as a function only of θ that involves logarithms, sine, and / or cosine (but no other functions).