MAT131 Fall 2022 Paper HW 7

Due the week of October 17 – October 21. For all problem sets, students are allowed to work together. However, the final answer you turn in must be based on your own understanding and must be in your words. Per university policy, all instances of suspected academic dishonesty will be referred to the academic judiciary.

Problem 1. Use the formula for the derivative of $\ln(x)$ using the method of inverse functions to compute the limit as h approaches 0 of $(1/h)\ln(1+h)$. Next consider the quantity $(1+h)^{1/h}$. Find a continuous function that transforms the limit of $(1/h)\ln(1+h)$ into $(1+h)^{1/h}$. Use this to evaluate the limit as h approaches 0 of $(1+h)^{1/h}$. Finally, let h be $1/n$ for a positive integer n, and expand $(1+(1/n))^n$ using the Binomial Theorem. For a given positive integer m, as n tends to ∞, what is the limit of the m^{th} summand of the Binomial Theorem expansion of $(1+(1/n))^n$? This gives a series of fractions that converges to e.

Problem 2. A wire is bent into the shape of a parabola $y = x^2$. A bead slides along the wire, falling along the parabola in the first quadrant towards the origin. At the moment when the bead is at coordinates (a, a^2), the distance from the bead to the origin is decreasing at a rate of b, i.e., increasing at a rate of $-b$. Compute the rates of change of both the x-coordinate and y-coordinate at that moment. Double-check your answer by comparing the ratio of these rates of change to the slope of the tangent line to the parabola at (a, a^2).