

MAT131 Fall 2022 Paper HW 6

Due the week of October 10 – October 14. For all problem sets, students are allowed to work together. However, the final answer you turn in must be based on your own understanding and must be in your words. Per university policy, all instances of suspected academic dishonesty will be referred to the academic judiciary.

Problem 1. Let e and p be positive real numbers with $0 < e < 1$. Let $r(\theta)$ be the function $ep/(1 - e \cos(\theta))$. For the curve with parametric equation $(x(\theta), y(\theta)) = (r(\theta) \cos(\theta), r(\theta) \sin(\theta))$, find a formula for the slope of the tangent line at the point $(x(\theta), y(\theta))$ as a fraction of simplified trigonometric equations in θ . Compute the two angles $-\pi/2 < \theta < \pi/2$ at which the tangent line is horizontal, and compute the distance between the y -coordinates of these two points (the “major axis”) as a simplified fraction in e and p .

Problem 2. Let $y = f(x)$ for the function $f(x) = (e^x - e^{-x})/(e^x + e^{-x})$ with domain $(-\infty, +\infty)$ and range $(-1, +1)$. Check that an inverse function is given by $x = g(y)$, $g(y) = \frac{1}{2} \ln((1 + y)/(1 - y))$. Check using derivatives of logarithm functions that $g'(y)$ equals $1/(1 - y^2)$. Next compute the derivative $f'(x)$ and use your formula to compute $1/f'(g(y))$. Check that this also simplifies to $1/(1 - y^2)$.