MAT131 Fall 2022 Paper HW 5

Due the week of October 3 – October 7. For all problem sets, students are allowed to work together. However, the final answer you turn in must be based on your own understanding and must be in your words. Per university policy, all instances of suspected academic dishonesty will be referred to the academic judiciary.

Problem 1. For each positive integer n, let f_n be a differentiable and nowhere zero function on an open interval (a, b). Iteratively apply the Product Rule to compute simplified formulas for the following in terms of f_1'/f_1, f_2'/f_2, f_3'/f_3, and f_4'/f_4.

$$\frac{1}{f_1 f_2}\frac{d(f_1 f_2)}{dx}, \quad \frac{1}{f_1 f_2 f_3}\frac{d(f_1 f_2 f_3)}{dx}, \quad \frac{1}{f_1 f_2 f_3 f_4}\frac{d(f_1 f_2 f_3 f_4)}{dx}.$$

Formulate a guess for

$$\frac{1}{f_1 f_2 \cdots f_{n-1} f_n}\frac{d(f_1 f_2 \cdots f_{n-1} f_n)}{dx},$$

as an expression in $f_1'/f_1, \ldots, f_n'/f_n$. Explain why your guess is correct.

Problem 2. For every differentiable function $f(x)$ on an interval (a, b) and every $c > 0$, the function $g(x) = f(cx)$ on $(a/c, b/c)$ is differentiable with derivative,

$$g'(x) = \lim_{h \to 0} \frac{f(cx + ch) - f(cx)}{h} = c \lim_{ch \to 0} \frac{f(cx + ch) - f(cx)}{ch} = cf'(cx).$$

Use this compute the derivative $d\sin(2x)/dx$, and simplify your answer to a polynomial expression in $\sin(x)$ and $\cos(x)$ using the double-angle formulas from trigonometry. Next, use the double-angle formula to first express as $\sin(2x)$ as a polynomial expression in $\sin(x)$ and $\cos(x)$, and then differentiate this expression using the Product Rule. Check that both methods give the same derivative $d\sin(2x)/dx$ as a polynomial expression in $\sin(x)$ and $\cos(x)$.