MAT131 Fall 2022 Paper HW 4

Due the week of September 19 – September 23. For all problem sets, students are allowed to work together. However, the final answer you turn in must be based on your own understanding and must be in your words. Per university policy, all instances of suspected academic dishonesty will be referred to the academic judiciary.

Problem 1. For every real number a, compute the derivative of $y = x^4$ at $x = a$ as a limit of a difference quotient. Show your work.

Problem 2. For every real number $a \neq 0$, for every nonnegative integer n, for the function $f(x) = x^n$, simplify the limit,

$$\lim_{q \to 1} \frac{f(q \cdot a) - f(a)}{(q \cdot a) - a}.$$

Evaluate your simplified limit using the formula for a finite geometric sum,

$$1 + q + q^2 + \cdots + q^{n-1} = \frac{q^n - 1}{q - 1}.$$

Use this to compute $f'(a)$ for $a \neq 0$.