

# MAT131 Fall 2022 Paper HW 2

Fall 2022

**Due the week of September 5 – September 9.** For all problem sets, students are allowed to work together. However, the final answer you turn in must be based on your own understanding and must be in your words. Per university policy, all instances of suspected academic dishonesty will be referred to the academic judiciary.

**Problem 1.** The following function has domain  $(-1, 0) \cup (0, \infty)$ ,

$$f(x) = \frac{-1 + \sqrt{1+x}}{x}.$$

Make a table of values of  $f(x)$  for  $x = \pm 1, \pm 0.1, \pm 0.01$ , and  $\pm 0.001$ . Use your table to **guess** a limiting value for  $f(x)$  as  $x \rightarrow 0$ . Finally, use difference-of-squares rationalization to find a continuous function  $g(x)$  whose domain contains  $(-1, \infty)$  such that  $f(x)$  equals  $g(x)$  on  $(-1, 0) \cup (0, \infty)$ . Use this to prove that your guess is correct.

**Problem 2.** Define a function  $h(x)$  on  $(-\infty, +\infty)$  by the following rule,

$$h(x) = \begin{cases} \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

Compute the limit as  $x$  approaches 0 of  $k(x) = xh(x)$ . Explain why it is invalid to simply compute the limit as  $k(0) = 0 \cdot h(0) = 0 \cdot 0 = 0$ ?