MAT 131 Calculus I
Stony Brook University
Exam 1, Wednesday 9/28/2021

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MAT 131 Midterm 1 Review

Exam Policy. Exam 1 will be held on Wednesday, September 28th, from 7:50 PM to 9:15 PM. The rooms are to be announced. The exam is closed book, closed notes, no electronic devices are allowed, and you need only bring a writing implement. You will write directly on the exam booklet. Scratch paper and a stapler will be provided.

Review Topics. Please be familiar with all of the following course outcomes / key skills.

- Definition, basic properties and graphs of elementary functions: powers, exponentials, logarithms, and trigonometric.
- The definition, basic properties and graphs of even and odd functions.
- The definition and meaning of increasing and decreasing for functions and graphs.
- Reflection, translation and scaling of graphs and the corresponding transformation of the functions.
- Definition, basic properties, and graphs of inverse functions. Computation of an inverse function.
- Intuitive definition, basic laws, and techniques for computing limits, one-sided limits, limits using the squeeze theorem, limits equal to infinity, and limits at infinity. Students are NOT expected to work with ϵ-δ notions of the limit on the midterm.
- Identifying all discontinuity points (both the location and type), the domain of a function, and all vertical and horizontal asymptotes. Application of these notions to curve-sketching.
- The statement of the Intermediate Value Theorem and its use in finding zeroes of functions.
- The definition of the derivative as the limit of a difference quotient, and methods for computing derivatives directly from the definition.
- Using the derivative to compute the equations of tangent lines.
- Using the rules of differentiation: the sum rule, the product rule, the quotient rule, the power rule, and derivatives of exponential functions.

Important Note. The best preparation is to understand the material from the textbook and the homework problems, both the assigned and unassigned problems. The exam will have some problems that test understanding of statements of propositions and theorems, but there will be an emphasis on applications of the results to computation.

Problem 1. In each of the following cases, determine whether the limit exists as a finite number, and say its value if it is defined. If the limit does not exist as a finite number, determine whether the limit is positive or negative infinity. If the limit does not exist as a finite number or as positive/negative infinity, explain why.

(a)

$$\lim_{x\to 0} f(x), \text{ where } f(x) = \begin{cases} \sqrt{x}, & x > 0 \\ -\sqrt{-x}, & x \leq 0 \end{cases}$$

(b)

$$\lim_{x\to 0} \frac{x + \mid x \mid}{x}.$$

(c)

$$\lim_{x\to 0} \frac{\sqrt{x^2}}{x}$$

(d)

$$\lim_{x\to 2} \frac{x^3 - 2x^2 - 4x + 8}{x^2 - 4}$$

(e)

$$\lim_{x\to 1} \frac{\ln(5^x)}{x}$$

(f)

$$\lim_{x\to 0^-} (3x + \sqrt{9x^2 + 6x})$$

(g)

$$\lim_{x\to 2} \frac{|x-2|}{x-2}$$

(i)

$$\lim_{x\to 1} \frac{x^2 - 4x + 3}{x^2 - 1}$$

(j)

$$\lim_{x\to 0} \frac{\cos x}{x}$$

(k)

$$\lim_{x\to 0^+} \frac{\cos(x)}{x}$$

(l)

$$\lim_{x\to 0} \sin(1/x)$$

(m)

$$\lim_{x\to 0} \frac{-1+2x^{-1}+3x^{-3}}{1+4x^{-3}}$$

(n)

$$\lim_{x\to 0} \frac{4^{1/x}}{2^{1/x}}$$

(o)

$$\lim_{x\to 0^+} \ln(x)$$

(p)

$$\lim_{x\to 0} \ln(|\ x\ |)$$

(q)

$$\lim_{x\to 2} \frac{x^2-5x+6}{x^2-4}$$

(r)

$$\lim_{x\to 3} \frac{x^2-6x+9}{2x-6}$$

(s)

$$\lim_{x\to 1} \frac{\ln(x)}{\ln(x^2)}$$

(t)

$$\lim_{x\to 0} \left(\frac{x+1}{x}+1+\frac{x-1}{x}\right)$$

(u)

$$\lim_{x\to 0} \left(\frac{1}{\frac{1}{x}-\frac{x^2+1}{x^3}}\right)$$

(v)

$$\lim_{x\to 0^+} \left(\sqrt{1+x^{-2}}-x^{-1}\right)$$

$$\lim_{x\to 0^-} \left(\sqrt{1+x^{-2}}-x^{-1}\right)$$

(w)

$$\lim_{x\to 0^+} \left(\sqrt{1+x^{-2}}+x^{-1}\right)$$

$$\lim_{x\to 0^-}\left(\sqrt{1+x^{-2}}+x^{-1}\right)$$

(x)

$$\lim_{x\to 0}\frac{x^9-1}{x-1}$$

(y)

$$\lim_{x\to 0}\frac{1}{\sin(x)}$$

$$\lim_{x\to 0}\frac{1}{|\sin(x)|}$$

(z)

$$\lim_{x\to 0}\ln(x^2)$$

$$\lim_{x\to 0^+}[\ln(x)]^2.$$

(α)

$$\lim_{x\to\infty}\frac{e^{x+1}-e^{x-1}}{e^{x+1}+e^{x-1}}$$

$$\lim_{x\to-\infty}\frac{e^{x+1}-e^{x-1}}{e^{x+1}+e^{x-1}}$$

(β)

$$\lim_{x\to\infty}e^{x+3}e^{2-2x}e^{x-5}$$

$$\lim_{x\to-\infty}e^{x+3}e^{2-2x}e^{x-5}$$

Problem 2 For the following function, state the domain, whether the function is even, odd or neither, and the location and type of any and all discontinuities.

$$f(x)=\frac{1-\sqrt{1-4x^2}}{2x}.$$

Problem 3 For each of the following functions, state the domain of the function, and the location and type of any and all discontinuities.

(a)

$$y=\frac{x}{\sqrt{x^2}-x}$$

(b)

$$y=\frac{x+2}{x^3+x^2-2x}$$

(c)

$$y = \frac{x}{1+\cos(x)}$$

Problem 4 Find the equations of all tangent lines to the graph of $y = x^2$ which contain the point $(3, 5)$. Please note this point is *not* on the graph. You may compute the derivative by any (correct) method you know.
Note. If this review problem is discussed in lecture, we will draw a picture. For a nice Java applet illustrating this problem, scan down to the "Archimedes triangle" section of this webpage on the parabola.

Problem 5 In each of the following cases, use the definition of the derivative as a limit of a difference quotient to compute the derivative of $y = f(x)$ at the point $x = a$. Then find the equation of the tangent line to the graph of $y = f(x)$ at the point $(a, f(a))$.

(a) $y = \sqrt{x+1}$ at $x = 3$

(b) $y = x + \frac{1}{x}$ at $x = -1$

(c) $y = x^3 + x^2$ at $x = -2$

(d) $y = \frac{x+1}{x-1}$ at $x = 0$

Problem 6 Use the definition of the derivative as a limit of a difference quotient to compute the derivative of $y = x^2 + \ln(1)\sin(x)$ at the point $x = 7$.

Problem 7 Use the definition of the derivative as a limit of a difference quotient to compute the derivative at $x = 0$ for the following function

$$y = \begin{cases} x^2, & x > 0 \\ 0, & x = 0 \\ -x^2, & x < 0 \end{cases}$$

Note. The derivative *is* defined at this point.

Problem 8 Determine whether or not the following function is continuous at $x = 0$.

$$y = \begin{cases} x^2\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Also determine whether or not the derivative of $y = f(x)$ is defined at $x = 0$. If it is defined, compute it. If it is not defined, explain why not.

Problem 9 In each of the following cases, use the definition of the derivative as a limit of a difference quotient to compute the *derivative function*.

(a)

$$f(x) = \frac{1}{x+3}, \text{ for } x \neq 3,\ f'(x) = ?$$

(b)

$$g(x) = 2x^2 - 4, \quad g'(x) = ?$$

(c)

$$f(x) = \sqrt{2x-7}, \quad f'(x) = ?$$

(d)

$$i(x) = \frac{1}{x+1} - \frac{1}{x-1}, \quad i'(x) = ?$$

Problem 10 Sketch the graph of a function $f(x)$ satisfying all of the following properties.

1. $\lim_{x\to 1^+} f(x) = 1$
2. $\lim_{x\to 1^-} f(x) = 0$
3. $f(1) = 1$
4. $\lim_{x\to -\infty} f(x) = 2$
5. $f(-2) = 4$
6. $\lim_{x\to -1^-} f(x) = -\infty$
7. $\lim_{x\to -1^+} f(x) = \infty$
8. $\lim_{x\to \infty} f(x) = -1$

Problem 11 In each of the following cases, say whether the statement is true or false for an everywhere continuous function $f(x)$ satisfying the stated hypothesis. If the statement is false, sketch a graph demonstrating it is false.

1. If $y = f(x)$ is increasing, then $y = -f(x)$ is increasing.
2. If $y = f(x)$ is increasing, then $y = -f(x)$ is decreasing.
3. If $y = f(x)$ is increasing, then $y = f(-x)$ is increasing.
4. If $y = f(x)$ is increasing, then $y = f(-x)$ is decreasing.
5. If $y = f(x)$ is even, it cannot be everywhere decreasing.
6. If $y = f(x)$ is odd, it cannot be everywhere decreasing.
7. A inverse function $y = f^{-1}(x)$ defined on an interval $[a, b]$ cannot be both increasing on (a, c) and decreasing on (c, b). (The inverse of a continuous function on an interval is also continuous.)

8. If there exists a function $y = g(x)$ defined on the set of all real numbers whose restriction to the range of $f(x)$ is an inverse of $f(x)$, then the domain of the inverse of $f(x)$ is the set of all real numbers and $g(x)$ satisfies the Horizontal Line Test on the domain of all real numbers.

Problem 12 In each of the following cases, compute the derivative using derivative rules (or limits of difference quotients, if you show all work). Explain what derivative rules you use at each step.

(a)

$$\frac{df(x)}{dx} = g(x), \quad \frac{d(f(x))^2}{dx} = ?$$

(b)

$$\frac{df(x)}{dx} = g(x), \quad \frac{d(f(x))^{-1}}{dx} = ?$$

(c)

$$\frac{d(x^n)}{dx} = ?, \quad n = 0, 1, 2, \ldots$$

(d)

$$\frac{d(x^{-n})}{dx} = ?, \quad n = 0, 1, 2, \ldots$$

(e)

$$f(x) = \frac{(x-2)x(x+2)}{(x-1)(x+1)}, \quad f'(x) = ?$$

(f)

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}, \quad f'(x) = ?$$

(g)

$$f'(x) = \cos(x), \quad \frac{d(f(x))^{-1}}{dx} = ?$$

(h)

$$\frac{d}{dx}(1 + x + x^2 + \cdots + x^n) = ?, \quad n = 2, 3, \ldots,$$