Name: ______________________________ Core Competency Exam A

You do not need to show work. Answer on the line.

_4_. **Problem 1** The equation $|x+1| = 2$ has: (1) no solutions, (2) a unique solution, (4) one positive and one negative solution, or (3) two positive solutions, (5) two negative solutions.

_1_. **Problem 2** The set of all real numbers where $|3x - 3| \le 6$ is:

$$(1)\ [-1,3],\ (2)\ (-1,3),\ (3)\ (-\infty,-1]\cup[3,\infty),\ (4)\ (-\infty,-1)\cup(3,\infty),\ \text{or}\ (5)\ (-3/3,9/3].$$

_3_. **Problem 3** The reflection through the origin of the graph of $y = f(x)$ is the graph of:

$$(1)\ y = -f(x),\ (2)\ y = f(-x),\ \text{or}\ \ (3)\ y = -f(-x).$$

_2_. **Problem 4** For the function $f(x) = 1/(2x-1)$, $x \ge 0$, the value $f(f(0))$ equals:

$$(1)\ \ -1,\ \ (2)\ \ -1/3,\ \ (3)\ \ 1,\ \ (4)\ \ 3,\ \text{or}\ \ (5)\ \ \text{undefined}.$$

_3_. **Problem 5** For the functions $f(x) = 1 - (\frac{1}{x})$, $x \ne 0$, and $g(x) = \frac{1}{x+1}$, $x \ne -1$, the composite function $f(g(x))$, $x \ne -1$, equals

$$(1)\ \ \frac{x}{2-x},\ \ (2)\ \ \frac{1}{(-1/x)+2},\ \ (3)\ \ -x,\ \ (4)\ \ \frac{1}{1/(x+1)}+1.$$

_1_. **Problem 6** The equation of the line with slope -2 containing the point $(x,y) = (1,1)$ is:

$$(1)\ \ y = -2x+3,\ \ (2)\ \ y-1 = 2(x-1),\ \ (3)\ \ y = -2x+1,\ \ \text{or}\ (4)\ \ y-(-2) = 1(x-1).$$

_4_. **Problem 7** The line containing the two points $(x,y) = (1,0)$ and $(x,y) = (2,-2)$ has equation

$$(1)\ \ y-0 = \frac{2-1}{-2-0}(x-1),\ \ (2)\ \ y-1 = \frac{-2-0}{2-1}(x-0),\ \ (3)\ \ y = 2x-2,\ \ \text{or}\ (4)\ \ y = -2x+2.$$

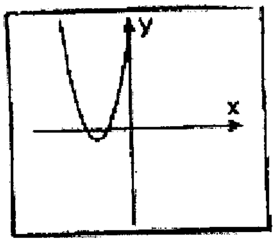
_3_. **Problem 8** The perpendicular line to $y = 3x+2$, containing the point $(1,1)$ has equation

$$(1)\ \ y = 3x-2,\ \ (2)\ \ y = (-1/3)x-1,\ \ (3)\ \ y = (-1/3)x+(4/3),\ \ \text{or}\ (4)\ \ y = (1/3)x+2/3.$$

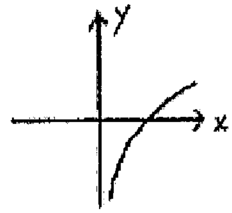
_1_. **Problem 9** The solutions of the quadratic equation $x^2 + 4x = 0$ are (1) $x = 0$ and $x = -4$, (2) $x = 0$ and $x = -2$, (3) $x = -4$ and $x = +4$, or (4) undefined.

_4_. **Problem 10** The parabola with equation $y = x^2 - 3x + 2$ satisfies $y > 0$ for x in

$$(1)\ \ [1,2],\ \ (2)\ \ (1,2),\ \ (3)\ \ (-\infty,1]\cup[2,\infty),\ \ (4)\ \ (-\infty,1)\cup(2,\infty).$$



PROBLEM 1



PROBLEM 9

Name: ______________________________ Core Competency Exam B

1. **Problem 1.** The displayed conic section is a: (1) parabola, (2) circle, (3) ellipse, or (4) hyperbola.

3. **Problem 2.** The number $(27)^{1/2}/\sqrt{9}$ equals:

(1) $3\sqrt{3}$, (2) 3, (3) $\sqrt{3}$, (4) $3^{-1/2}$, or (5) 1.

5. **Problem 3.** The function $(x-x^2)/\sqrt[3]{x^4}$, $x>0$, equals:

(1) $(1-x)/\sqrt{x}$, (2) $\sqrt[3]{(x^3-x^6)/x^4}$, (3) $\sqrt[3]{(x^3/x^4)}+\sqrt[3]{(x^6/x^4)}$, (4) $x^{-2/3}-x^2$, or (5) $x^{-1/3}-x^{2/3}$.

4. **Problem 4.** The function $(xy^2)^3/(xy)^4$, $x>0$, $y>0$, equals:

(1) $(x^3y^2)/(x^3y^4)$, (2) y^2, (3) $x^{-2}y$, (4) $x^{-1}y^2$, or (5) $(3xy^2)/(4xy)$.

2. **Problem 5.** The function $(x^2+1)^2-(x^2+1)$ equals

(1) $(x^4+1)-(x^2+1)$, (2) x^4+x^2, (3) x^4-x^2, (4) $((x^2+1)-x)((x^2-1)-x)$, or (5) x^2.

4. **Problem 6.** The solution x of the equation $5(2^x)=80$ is:

(1) $\log_2(80)/\log_2(5)$, (2) 16, (3) 3, (4) 4, or (5) $\log_{80}(80)-\log_{80}(5)$.

4. **Problem 7.** The function $\log_2(4\sqrt{x})$, $x>0$, equals:

(1) $1+\log_4(\sqrt{x})$, (2) $\frac{1}{2}\log_2(2x)$, (3) $\frac{1}{2}\log_4(16x)$, (4) $2+\frac{1}{2}\log_2(x)$,, or (5) $\sqrt{(\log_2(16x)}$.

3. **Problem 8.** The solution $x>0$ of the equation $\log_2(4x^3)=11$ is:

(1) 2048, (2) $2^{11}/4$, (3) 8, (4) 3, or (5) $\sqrt{2^{11}/4}$.

4. **Problem 9.** The displayed graph might be the graph of the function $f(x)=$

(1) $5(2^x)$, (2) $3(1/2)^x$, (3) $-2(3^x)$, or (4) $2\log_3(x)$.

1. **Problem 10.** For the unique real numbers $x>0$, $y>0$ with $\log_2(x)=2$ and $\log_2(y)=1/3$, the expression $\log_2(x^3y)$ equals:

(1) 19/3, (2) $4^3\sqrt[3]{2}$, (3) 3, (4) $3\log_2(x)+\log_2(y)$, or (5) $2^3\cdot(1/3)$.

Mastery Exam. Show all Work.

Name: ______________________________ **Problem 1:** ____________ /35

Mastery Problem 1(35 points) For all parts of this problem, $f(x)$ equals $5^{2x} - 6(5^x) + 9$. Show all work.

(a)(2 points) Find the domain of f, i.e., the maximal set of real numbers for which the expression is defined as a real number. Express your answer using interval notation.

f defined for every real x.

$\boxed{\text{Domain} = (-\infty, +\infty)}$

(b)(10 points) Find the two real numbers c such that $f(c)$ equals 4.

$f(x) = (5^x)^2 - 6(5^x) + 9 = (5^x - 3)^2.$

$(5^c - 3)^2 = 4, \quad 5^c - 3 = \begin{cases} +2 \\ -2 \end{cases}, \quad 5^c = \begin{cases} 5 \\ 1 \end{cases}$

$\boxed{c = \begin{cases} 1 \\ 0 \end{cases}}$

Name: ____________________ **Problem 1 continued.**

(c)(3 points) Find the range of f. Express your answer in interval notation.

$f(x) = (5^x - 3)^2 \geqslant 0$. For $x \in [\log_5(3), +\infty)$, $f(x) \in [0, +\infty)$.

$\boxed{\text{Range} = [0, +\infty)}$

(d)(15 points) Now restrict $f(x)$ to the domain $[1, \infty)$. State the corresponding **range** of the restricted function, and find a formula for the inverse $f^{-1}(x)$ of this restricted function.

$f(1) = (5^1 - 3)^2 = 2^2 = 4$. $\boxed{\text{Range} = [4, +\infty)}$

$y = (5^x - 3)^2$, $\sqrt{y} = 5^x - 3$, $5^x = 3 + \sqrt{y}$, $\log_5(5^x) = \log_5(3 + \sqrt{y})$

$x = \log_5(3 + \sqrt{y})$. $\boxed{f^{-1}(x) = \log_5(3 + \sqrt{x})}$

(e)(5 points) Rewrite your answer from (d) using only natural logarithms, i.e., $\ln(u) = \log_e(u)$. Your answer should involve only addition, subtraction, multiplication, division, powers, roots and the natural logarithm – **no** exponential functions and **no** logarithms with a base **different** from e.

$\log_5(u) = \dfrac{\log_e(u)}{\log_e(5)}$. $\boxed{f^{-1}(x) = \dfrac{\ln(3 + \sqrt{x})}{\ln(5)}}$.

Name: ______________________________ **Problem 2:** ____________ /30

Mastery Problem 2(30 points) Perform the following polynomial computations. Express your answers in the form $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ for positive whole numbers n and real numbers $a_0, a_1, a_2, \ldots, a_n$. Show all work.

(a)(15 points) Beginning with the polynomial function $f(x) = x^2 + 2x$, find a polynomial expression for the function

$$g(x) = \frac{f(x+3) - f(3)}{x}, \quad x \neq 0.$$

$f(x+3) = (x+3)^2 + 2(x+3) = (x^2+6x+9) + (2x+6) = x^2+8x+15$

$f(3) = 3^2+2\cdot 3 = 9+6 = 15$

$f(x+3) - f(3) = x^2+8x+15-15 = x^2+8x$

$\frac{f(x+3)-f(3)}{x} = \frac{x^2+8x}{x} = \boxed{x+8}$

(b)(15 points) For $h(y) = y^3 - 3y$ and for $k(x) = 2x + 1$, find $h(k(x))$.

$(2x+1)^3 = 2^3\cdot x^3 + 3\cdot 2^2\cdot x^2 + 3\cdot 2x + 1 = 8x^3 + 12x^2 + 6x + 1$

$-3(2x+1) = -6x-3.$

$(2x+1)^3 - 3(2x+1) = 8x^3+12x^2+6x+1-6x-3$

$= \boxed{8x^3+12x^2-2}$

Name: ______________________ **Problem 3:** __________ /35

Mastery Problem 3(35 points) A sample of bacteria grows from 1 gram to 4 grams in 16 hours. Assume that the bacteria mass follows an exponential growth model. Do all of the following. You may follow whatever order you prefer, but please show all work and indicate clearly your answers to each part.

(a)(15 points) Write the formula for the mass $a(t)$ of the bacteria sample after time t. Please include appropriate units in your answer (the same units used above), and leave no undefined constants in your final answer.

(b)(10 points) Find the doubling time for the bacteria sample.

(c)(10 points) Find the total amount of time needed for the bacteria sample to grow from 1 gram to 32 grams.

$a(t) = a_0 (2)^{t/d}$, a_0 = initial mass $= 1g$

d = doubling time.

$4g = a(16\text{ hrs}) = 1g \cdot (2)^{16\text{hrs}/d}$, $(2)^{16\text{hrs}/d} = 4 = 2^2$

$\frac{16\text{ hrs}}{d} = 2$, (b) $\boxed{d = 8\text{ hrs}}$

(a) $\boxed{a(t) = (1g)\,(2)^{t/8\text{hrs}}}$

(c) $32g = a(t) = (1g)\,(2)^{t/8\text{hrs}}$. $(2)^{\frac{t}{8\text{hrs}}} = 32 = 2^5$

$\frac{t}{8\text{ hrs}} = 5$, $t = 5 \cdot 8\text{ hrs}$, $\boxed{t = 40\text{ hours}}$