MAT 614 Assorted Problems

These are some assorted problems that I have collected.

Problems.

Problem 1. Can the valuative criterion of properness be checked on a dense open subset of the domain? For a finite type morphism to an affine scheme, to check the valuative criterion of separatedness, resp. for properness, does it suffice to work with morphisms of spectra of valuation rings mapping the generic point into a specified dense open subset of the domain?

Problem 2. Can a G-torsor (for the étale topology) on the complement of a divisor on a surface be extended away from finitely many points when G is simply-connected and semisimple?

Problem 3. Is every smooth morphism whose geometric fibers are affine n-space locally for the étale topology equal to projection from a product with affine n-space?

Problem 4. For a positive integer n, for a finite group G, does there exist a Calabi-Yau variety of dimension n that has a fixed-point-free action of G by biholomorphisms?

Problem 5. For a quasi-projective morphism that is flat, is the pushforward of the structure sheaf also flat?

Problem 6. Which X are "rigid in the large", i.e., every smooth, projective morphism over a connected target having one geometric fiber isomorphic to X has every geometric fiber isomorphic to X?'

Problem 7. Is the following $(de)\times(det)$-determinant polynomial in variables $(x_i, y_j)_{i=1,\dots,de}$ irreducible where the matrix has entry $x_i^\ell y_i^m$ in row-i and column-$md+\ell+1$ for all $0 \leq \ell < d$ and $0 \leq m < e$?

Problem 8. Given torsors over Spec of a square-zero extension of a ring for an fppf group scheme G, given a morphism of the G-torsors modulo the square-zero ideal, what is the obstruction to lift this to a morphism of G-torsors over Spec of the entire ring?

Problem 9. What is the smallest possible dimension of an affine variety that admits an fppf morphism to projective n-space that is fppf locally projection from a product variety?

Problem 10. Does every smooth, proper stack over Spec of a field admit a surjective morphism from a scheme that is smooth and proper?

Problem 11. For a very general principally polarized Abelian variety of dimension g, what is the minimal degree of a finite, flat morphism to projective g-space?

Problem 12. When is a smooth, irreducible curve in affine 3-space a complete intersection of two hypersurfaces?

Problem 13. What is the Brauer group of the stack of principally polarized Abelian varieties of dimension g, resp. the stack of smooth, projective curves of genus $g > 1$?

Problem 14. Do the global sections of a locally free sheaf on a quasi-projective scheme form a projective module over the global sections of the structure sheaf?

Problem 15. Does there exist a pencil of plane curves of degree > 2 that has only two singular members?

Problem 16. Is every smooth, projective curve birational to an affine curve in the plane of the form $f(x) = g(y)$?

Problem 17. Over a characteristic 0, non-algebraically closed field k, does Lüroth's theorem hold, i.e., is every k-unirational smooth, projective surface k-rational?

Problem 18. For the codimension-3 intersection in $\mathbb{P}^3 \times \mathbb{P}^3$ of general hypersurfaces of degrees $(1, 1)$, $(1, 1)$ and $(2, 2)$, how many fibers of (one of the two) projections have dimension > 0?

Problem 19. For a normal variety that admits a log resolution, given a tangent vector field on the smooth locus, does this lift to a logarithmic tangent vector field on the resolution with log poles on the exceptional locus?

Problem 20. Is every torus-invariant, equidimensional, reduced closed subset of projective space a specialization of smooth subvarieties of projective space?