MAT 589 – INTRODUCTION TO ALGEBRAIC GEOMETRY – SPRING 2020

Class webpage: `http://www.math.stonybrook.edu/~jstarr/M589s25/index.html`
Class Meetings: Lecture is held Mondays and Wednesdays, 9:30 AM – 10:50 AM in Math Tower 4-130.
Course Announcements: Announcements about the course will be posted on the class webpage. Please check the site regularly for announcements (which will also be given in lecture and/or in recitation).
Course Description: The description in the graduate bulletin: This course offers a systematic introduction to algebraic geometry, from a modern, scheme-theoretic perspective.

Prerequisites: MAT 536, or permission of instructor. Familiarity with material covered by MAT 545 would be helpful, but is not required.

Text: Although it is not listed as a required textbook, this semester the course will closely follow the textbook by Hartshorne.
Robin Hartshorne, *Algebraic geometry*, Springer.

Course Learning Objectives: The course learning objectives include the following. Each of these is an important learning objective for *all* advanced mathematics and applied mathematics courses. Each is amplified with specific examples.

• **Acclimate to New Mathematics.** Gain familiarity with a new mathematical idea (be it a definition, a result, an algorithm, etc.) through examples, through basic results that involve that idea, and through deeper results that reflect the significance of the idea. **Example.** The definition of scheme and morphism of scheme using the language of sheaves.

• **Apply and Model.** Understand how an abstract notion or result can lead to an algorithm or computation arising in a context different from the original notion or result. Understand the necessary hypotheses and limitations of that model. **Example.** Using the Koszul complex from commutative ring theory to compute sheaf cohomology of a quasi-coherent sheaf on an affine scheme.

• **Specialize.** Pass from general theorems, definitions, and methods to specific examples. Be able to compute with those examples. **Example.** The general Koszul complex specializes to the Euler sequence computing the sheaf of differentials on projective space.

• **Generalize.** Understand examples of ideas, constructions and arguments originally developed in one context yet that extend to another context. **Example.** Notions related to a covering, e.g., glueing for sheaves, apply to more general morphisms such as torsors for the multiplicative group.

• **Prove.** For a conjectured result, often expected from examples, heuristics and other indirect evidence, rigorously prove the result using techniques such as proof by induction, proof by contradiction, proof by cases, and more advanced proof techniques. **Example.** Serre duality is strongly suggested by the computation of cohomology of invertible sheaves on projective space, but the rigorous proof also involves results about Ext and Grothendieck's formalism of universal delta functors.

Course Outcomes / Key Skills: The course outcomes / key skills include the following.

(1) Know about sheaves, morphisms between sheaves, and functors between categories of sheaves.
(2) Know about schemes, morphisms, and properties of morphisms, including which properties are local on the domain / target, which properties are compatible with composition, which properties are compatible with base change, etc.
(3) Know the valuative criteria for separatedness and properness, and be able to apply these in examples, e.g., for the morphism from a relative Proj to the target scheme.
(4) Know Chow's Lemma and be able to apply this, e.g., to prove the valuative criterion of properness.

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(5) Know the basic results about quasi-coherent / coherent sheaves on Noetherian schemes, including the equivalence to the category of modules / finitely generated modules on affine schemes, and the equivalence to the (localization at isomorphisms in high degree) to the category of graded modules / finitely generated graded modules on projective schemes.
(6) Know the proof that the direct image of a coherent sheaf via a projective morphism is again a coherent sheaf.
(7) Know the definition and basic properties of sheaf cohomology.
(8) Know the proof of vanishing of higher sheaf cohomology for quasi-coherent sheaves on affine schemes.
(9) Know the computation of sheaf cohomology of invertible sheaves on projective space.
(10) Know the proof and applications of Serre duality, e.g., to the proof of the Theorem of Enriques-Severi-Zariski on connectedness of ample divisors of positive dimension in a projective variety.

Instructor: The instructor is Jason Starr, Math Tower 4-108, E-mail: jstarr@math.stonybrook.edu

Office Hours: Office hours for Jason Starr are scheduled as follows.

- Monday 12 noon – 1 PM in Math Tower 4-108 (advising).
- Wednesday 11 AM – 12 noon in Math Tower 4-108.
- Thursday 12 noon – 1 PM in Math Tower 4-108.

Grading System: Grading is based on class participation, a final report and weekly problem sets.

Problem Sets: There will be 14 weekly problem sets assigned. All registered students are required to submit careful, complete solutions for at least 3 of these problem sets. These problem sets are primarily to help students master the material. The problems are drawn from various sources, including the recommended textbooks (but students are not required to purchase these textbooks).

IN LIGHT OF COURSE CHANGES, HOMEWORK WILL BE ACCEPTED AS A SCANNED DOCUMENT. THE SCHEDULE OF WEEKLY PROBLEM SETS IS MODIFIED BELOW. STUDENTS ARE NOW ALLOWED TO TURN IN PROBLEM SETS LATE, UP TO THE FINAL DAY OF LECTURES.

- Problem Set 1 is due 2/5/2025.
- Problem Set 2 is due 2/12/2025.
- Problem Set 3 is due 2/19/2025.
- Problem Set 4 is due 2/26/2025.
- Problem Set 5 is due 3/5/2025.
- Problem Set 6 is due 3/12/2025.
- Problem Set 7 is due 3/26/2025.
- Problem Set 8 is due 4/2/2025.
- Problem Set 9 is due 4/9/2025.
- Problem Set 10 is due 4/16/2025.
- Problem Set 11 is due 4/23/2025.
- Problem Set 12 is due 4/30/2025.
- Problem Set 13 is due 5/7/2025.
- Problem Set 14. This problem set is NOT TO BE HANDED IN. It is a problem set that corresponds to final material. Please look it over.

Required Syllabi Statements: The University Senate Undergraduate and Graduate Councils have authorized that the following required statements appear in all teaching syllabi (graduate and undergraduate courses) on the Stony Brook Campus.

Student Accessibility Support Center (SASC) statement: If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact the Student Accessibility Support

Center, Stony Brook Union Suite 107, (631) 632-6748 or at sasc@Stonybrook.edu. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential.

Student Accessibility Support Center

Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and SASC. For procedures and information go to the following website:

Evacuation guide for people with physical disablilities

Academic integrity statement: Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty is required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at

Student Accessibility Support Services

Critical incident management: Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn. Further information about most academic matters can be found in the Undergraduate Bulletin, the Undergraduate Class Schedule, and the Faculty-Employee Handbook.