Problem Set 11

Disclaimer For open-ended problems, part of the problem is to give a precise formulation. Especially for the problems in Part II, you should do as much of the problems as is useful to you. For each problem, it is important you understand how to verify all details. However, if you are pressed for time, you may write-up only the most important steps, instead of every detail.
Late homework policy. Late work will be accepted only with a medical note or for another approved reason.

Cooperation policy. You are strongly encouraged to work with others, but the final write-up must be entirely your own and based on your own understanding.

Part I. These problems are from the textbook. You are expected to read *all* the problems from the sections of the textbook covered that week. You are asked to write-up and turn-in only the problems assigned below.

Part II. These problems are not necessarily from the textbook. Often they will be exercises in commutative algebra, category theory, homological algebra or sheaf theory.

Part I(25 points)

- **(a)** (10 points) p. 222, Section III.4, Problem 4.1 See note below
- **(b)** (15 points) p. 222, Section III.4, Problem 4.2

Note (a). You are free to use the Noetherian hypothesis, but it is not necessary. For a slightly different approach, consider a flasque resolution of the quasi-coherent sheaf $\mathcal{F}$ on X and form the pushforward by f_*. This is a complex of $\mathcal{O}_Y$-modules. By what was proved in lecture, the pushforward of a flasque sheaf is flasque. What does Thm. III.3.5 tell you about exactness of the pushforward complex? (We will soon remove the Noetherian hypothesis from Thm. III.3.5.)

Part II(25 points)

Problem 1(10 points) Do Problem 4.5 of §III.4 on p. 224. You do not have to write up the following, but do think about it: Since the group of Cartier divisors is $H^0(X, \mathcal{K}^*/\mathcal{O}^*)$, what can you say about the map from the group of Cartier divisors to the Picard group? Using this, is there another way of thinking about Remark II.6.14.1 and Prop. II.6.15?

Problem 2(15 points) Let X be the scheme from Exercise II.6.7 on p. 148. Recall there is an isomorphism of $\text{Pic}(X)$ with the multiplicative group k^*. For every $\lambda \in k^*$, denote by $\mathcal{L}_\lambda$ an invertible sheaf on X whose class in $\text{Pic}(X)$ corresponds to λ. Using Čech cohomology or any other result about cohomology you like, compute the sequence of all sheaf cohomology groups $\{H^p(X, \mathcal{L})\}_{p=0,1,\dots}$. There will be two different behaviors for this sequence depending on λ.

Extra credit(5 points) Does (either direction of) Chevalley's theorem hold if one replaces the word "affine" by "quasi-affine and quasi-compact"? The claim on the earlier version of this problem set was wrong. (Hint: One direction does hold, and the other does not.)