SOLUTIONS TO MIDTERM

Midterm 2

MAT 118

April 2, 2009

Name: (please print)		**ID #:**
Your recitation: Rec 1, W 5:20-6:15	Rec 2, Tu 11:20-12:15 (circle your recitation)	Rec 3, Th 12:50-1:45

	1 15 pts	2 15 pts	3 15 pts	4 15 pts	5 15 pts	Total 75 pts
Grade						

Directions: There are 5 problems on 6 pages (including this one) in this exam. Please make sure you have all the pages.

Do all of your work in this exam booklet, and cross out any work that should be ignored. **Show your reasoning and computations — not just the answer.**

You will need a scientific calculator. You can also use a single letter size sheet of paper with the most important formulas or other information you prepared at home. You can not use any other materials, including books.

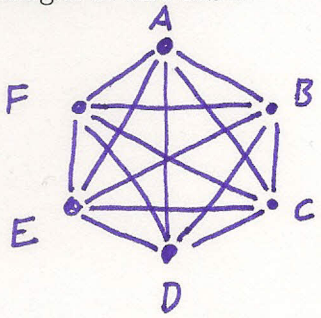
You have 1 hr 20 minutes.

Good luck!

DO NOT OPEN THE EXAM
UNTIL INSTRUCTED BY THE PROCTOR!

1. The following mileage chart shows distances between 6 cities: A, B, C, D, E, F. Use Kruskal agloritham to find the minimal spanning tree connecting the cities; circle in the chart the roads (edges) you have included in the MST. Compute the length of this MST.

	A	B	C	D	E	F
A	*	533	798	1068	1361	772
B	533	*	656	713	1071	802
C	798	656	*	447	592	248
D	1068	713	447	*	394	695
E	1361	1071	592	394	*	760
F	772	802	248	695	760	*

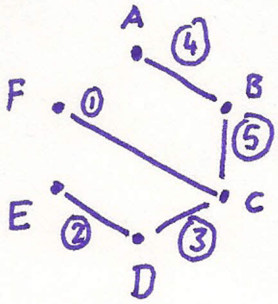


Edges by Cost (incr.)

Edge	Cost	
C-F	248	1
D-E	394	2
C-D	447	3
A-B	533	4
C-E	592	Loop w/ 2, 3
B-C	656	5
D-F	695	
B-D	713	
E-F	760	
A-F	772	
A-C	798	
B-F	802	
A-D	1068	
B-E	1071	
A-E	1361	

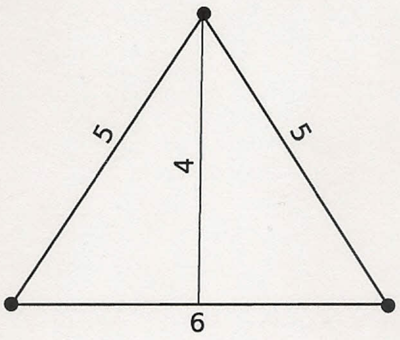
Minimal Spanning Tree

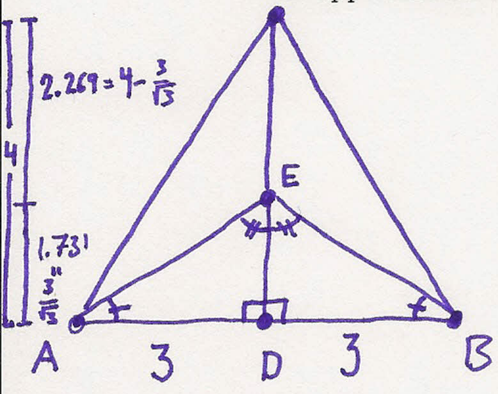
C-F	248
D-E	394
C-D	447
A-B	533
B-C	656
	2,282



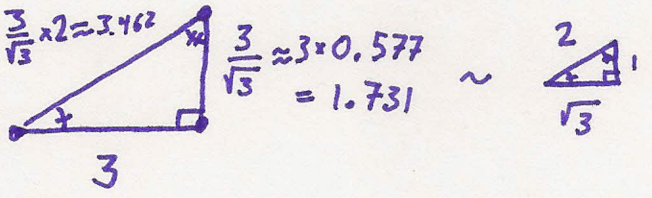
Total distance: 2,282 miles

2. Three points in a plane form an isosceles triangle with sides 6, 5, and 5 cm (see figure). The altitude of this triangle is 4 cm. Find the shortest network connecting these points (you will need to add a Steiner point), and compute its length, writing the answer both in symbolic form (e.g., $\sqrt{3}+1$) and as approximate value rounded to 3 decimal places.

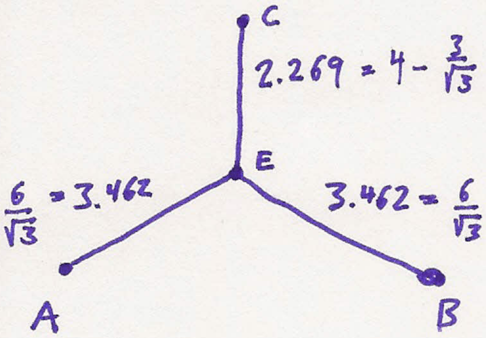




$\sqrt{3} \approx 1.732$, $\frac{1}{\sqrt{3}} \approx 0.577$



30-60-90 Triangle



① The triangle ΔADE is a 30-60-90 triangle, thus has legs in proportion to $1 : \sqrt{3}$ with hypotenuse in proportion to the shorter leg as $2:1$. Since the longer leg is 3, the shorter leg is $\frac{3}{\sqrt{3}} \approx 1.731$ ($\sqrt{3}$), and the hypotenuse is $\frac{2 \times 3}{\sqrt{3}} \approx 3.462$.

② Since $|DE| + |EC|$ equals $|CD| = 4$, and since $|DE|$ equals $\frac{3}{\sqrt{3}}$, it follows that $|EC|$ equals $4 - \frac{3}{\sqrt{3}} \approx 2.269$.

③ The total length of the shortest network is $|AE| + |BE| + |CE|$

$= \frac{6}{\sqrt{3}} + \frac{6}{\sqrt{3}} + 4 - \frac{3}{\sqrt{3}} = 4 + \frac{9}{\sqrt{3}} = \underline{4 + 3\sqrt{3}}$, which is approximately $\underline{9.196}$.

3. A nuclear power plant produces 12 pounds of radioactive waste every month, which must be stored in a special tank which can hold 500 pounds of the glowing goo. On January 1 2006, there were 25 pounds of waste in the tank (I guess it came with one pound for good luck). Let P_n represent the amount of radioactive waste in the tank after n months.

(a) Write an expression (either explicit or recursive) for P_n

(b) When will the tank be full?

Months	0	1	2	3	...	n
Waste (pounds)	25 $=25+\underline{0}\cdot 12$	$25+12=37$ $=25+\underline{1}\cdot 12$	$49=37+12$ $=25+\underline{2}\cdot 12$	$61=49+12$ $=25+\underline{3}\cdot 12$	...	$25+\underline{n}\cdot 12$

(a) This is an arithmetic sequence (or arithmetic progression) with $P_{n+1} = P_n + 12$. As illustrated by the table, there is an explicit expression $P_n = 25 + 12n$.

~~(b) Consider the sum of the first n terms of this arithmetic sequence~~

$$\cancel{\underbrace{\underbrace{\underbrace{\overset{25}{P_0}}_{1\text{ term}} + \overset{25+12}{P_1}}_{2\text{ terms}} + \overset{25+2\cdot 12}{P_2}}_{3\text{ terms}} + \cdots + \overset{25+(n-1)12}{P_{n-1}}}_{n\text{ terms}}}$$

~~The average of the first & last terms is~~ $\cancel{\frac{1}{2}(P_0 + P_{n-1}) =}$

(b) The tank will be full as soon as $P_n \geq 500$,

i.e. $25 + 12n \geq 500$

$12n \geq 475$

$n \geq \frac{475}{12} = \underline{39\frac{7}{12}}$. Thus in month $n = 40$, the tank will overflow.

4. In the country of Batavia, inflation is 50% a year: if an item cost 1 bat (local currency unit) in the beginning of a year, it will cost 1.5 bat at the end of the year. If a loaf of bread cost 3 bats now, how much will it cost 10 years from now? Round the answer to the nearest whole number.

n	0	1	2	3	4	5	6	7	8	9	10	11
2^n	1	2	4	8	16	32	64	128	256	512	1,024	2,048
3^n	1	3	9	27	81	243	729	2,187	6,561	19,683	59,049	177,147

Inflation after n years is $(1.5)^n = \left(\frac{3}{2}\right)^n = \frac{3^n}{2^n}$. Thus the future cost F of an object with initial cost P will be $F = P \cdot \frac{3^n}{2^n}$ after n years.

So one loaf of bread costs $3 \cdot \frac{3^{10}}{2^{10}} = \frac{3^{11}}{2^{10}} = 172\frac{919}{1024} \approx \boxed{173 \text{ bats}}$.

5. Mr. A and Mr. B both opened college savings account in a certain bank. The bank offers 6% annual interest rate; the interest is compounded monthly (assume all months have the same duration). Both accounts were opened on Jan 1, 2002. Mr. A deposits \$50 every month, starting Jan 1, 2002. Mr. B deposits \$7,000 when opening the account, but does not add anything in the following years. How much money will each of them have on Jan 1, 2010?

Monthly interest rate $= \frac{APR}{12} = \frac{0.06}{12} = 0.005$, $\underline{p = 0.005}$.

8 years = 96 months, Number of months : $\underline{n = 96}$.

Monthly installment by A: $\underline{M = \$50}$, Principal investment by B: $\underline{P = \$7{,}000}$.

$F_{A,n}$ = Value of A's investment after n months, $F_{B,n}$ = Value of B's investment after n months

Months	F_B	F_A
0	$P\cdot(1+p)^0$	$M(1+p)^0$
1	$P\cdot(1+p)^1$	$M(1+p)^1 + M(1+p)^0$
2	$P\cdot(1+p)^2$	$M(1+p)^2 + M(1+p)^1 + M(1+p)^0$
⋮		
n-1	$P\cdot(1+p)^{n-1}$	$M\cdot(1+p)^{n-1} + M\cdot(1+p)^{n-2} + \cdots + M\cdot(1+p)^1 + M\cdot(1+p)^0$
n	$P(1+p)^n$	$M(1+p)^n + M\cdot(1+p)^{n-1} + \cdots + M\cdot(1+p)^2 + M\cdot(1+p)^1$

Monthly Installment

. . .

← $\underline{\text{No monthly installment}}$ in final month.

So $F_{B,96} = P(1+p)^{96} = \$7{,}000\,(1.005)^{96} \approx \$7{,}000 \times 1.614 = \boxed{\$11{,}299}$.

Also $F_{A,96} = M(1+p)^1\left[\frac{(1+p)^n - 1}{(1+p) - 1}\right] = M(1+p)\left[\frac{(1+p)^n - 1}{p}\right] = \$50\cdot(1.005)\left[\frac{(1.005)^{96} - 1}{0.005}\right]$

$\approx \$50 \times (1.005) \times (122.83) = \$50 \times 123.44 = \boxed{\$6{,}172}$