

This is an amazingly well-written article of the highest mathematical quality! The authors study a fairly general class of operators which will play a central role in the development of Floer homology. In fact, the long-term goal of the authors is to establish a new and ground-breaking approach to Floer theory.

This article lays the groundwork for index calculations via the spectral flow. The main contribution is the development of this theory for operators defined on finite intervals. The approach via interpolation theory and Hilbert scales is entirely new; however, the authors generously (!) attribute their spectral flow approach to Hofer-Wysocki-Zehnder. The classical approach (under stronger assumptions) is due to Robbin-Salamon and only works for operators defined on the real line with appropriate asymptotic boundary conditions. The first main contribution is to identify appropriate boundary conditions for finite intervals, in order to obtain Fredholm operators. The approach by the authors is new, even for the case of the real line.

The article is extremely well written! I tried to find typos and simply couldn't. The authors also provide an excellent introduction and come in an informative but concise way to their main result in Theorem A. The main part of the article is basically devoted to the proof of Theorem A, after establishing the necessary background. For completeness, the authors also provide various additional material in appendices A -- E, which I found very helpful.

I do have the two following minor remarks for the authors:

- 1) In 1.2 concerning "Motivation and general perspective," you basically send the reader to another article. I think it would be nice to include a bit more context also directly in this article.
- 2) You use very nicely colors and shades, in particular in the proofs. However, especially in a black/white printed version, some of the "light gray" colors become almost invisible. Maybe a "print-version" where all/some colors are set to black would be helpful.