MAT 322/523 ANALYSIS IN SEVERAL DIMENSIONS
HOMEWORK 3

DUE: MONDAY FEBRUARY 12, 12:00PM

- Each problem is worth 10 points.
- Submit the homework via Gradescope.
- Only submit problems 2, 3 and 4.

1. (Munkres §6.1) Show that the function $f(x, y) = |xy|$ is differentiable at $\mathbf{0}$, but is not of class C^1 in any neighborhood of $\mathbf{0}$.

2. (Munkres §6.4) Show that if $A \subset \mathbb{R}^m$ and $f\colon A \to \mathbb{R}$, and if the partials $D_j f$ exist and are bounded in a neighborhood of $\boldsymbol{a}$, then f is continuous at $\boldsymbol{a}$.

3. (Munkres §6.5) Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the equation

$$f(r, \theta) = (r \cos\theta, r \sin\theta).$$

It is called the **polar coordinate transformation**.

(a) Calculate Df and $\det Df$

(b) Sketch the image under f of the set $S = [1, 2] \times [0, \pi]$. [*Hint: Find the images under f of the line segments that bound S.*]

4. (Munkres §7.2) Let $f\colon \mathbb{R}^2 \to \mathbb{R}^3$ and $g\colon \mathbb{R}^3 \to \mathbb{R}^2$ be given by the equations

$$f(\boldsymbol{x}) = (e^{2x_1+x_2}, 2x_2 - \cos x_2, x_1^2 + x_2 + 2)$$
$$g(\boldsymbol{x}) = (3y_1 + 2y_2 + y_3^2, y_1^2 - y_3 + 1).$$

(a) If $F(\boldsymbol{x}) = g(f(\boldsymbol{x}))$, find $DF(\mathbf{0})$. [*Hint: Don't compute F explicitly.*]

(b) If $G(\boldsymbol{y}) = f(g(\boldsymbol{y}))$, find $DG(\mathbf{0})$.

5. (Munkres §7.3) Let $f\colon \mathbb{R}^3 \to \mathbb{R}$ and $g\colon \mathbb{R}^2 \to \mathbb{R}$ be differentiable. Let $F\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by the equation

$$F(x, y) = f(x, y, g(x, y)).$$

(a) Find DF in terms of the partials of f and g.

(b) If $F(x, y) = 0$ for all (x, y), find $D_1 g$ and $D_2 g$ in terms of the partials of f.