MAT 322/523 ANALYSIS IN SEVERAL DIMENSIONS
HOMEWORK 11

DUE: MONDAY APRIL 29, 12:00PM

- Each problem is worth 10 points.
- Submit the homework via Gradescope.
- Only submit problems 1, 2 and 4.

1. (Munkres §32.3) In $\mathbb{R}^3$, let
$$\omega = xydx + 2zdy - ydz.$$
Let $\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ be given by the equation
$$\alpha(u,v) = (uv, u^2, 3u+v).$$
Calculate $d\omega$, $\alpha^*\omega$, $\alpha^*(d\omega)$ and $d(\alpha^*\omega)$ directly.

2. (Munkres §33.1) Let $A = (0,1)^2$. Let $\alpha : A \to \mathbb{R}^3$ be given by the equation
$$\alpha(u,v) = (u, v, u^2+v^2+1).$$
Let Y be the image set of α. Evaluate the integral $\int_{Y,\alpha} x_2 dx_2 \wedge dx_3 + x_1 x_3 dx_1 \wedge dx_3$.

3. (Munkres §33.3)

(a) Let A be the open unit ball in $\mathbb{R}^2$. Let $\alpha\colon A \to \mathbb{R}^3$ be given by the equation
$$\alpha(u,v) = \left(u, v, \sqrt{1-u^2-v^2}\right).$$
Let Y be the image set of α. Evaluate the integral
$$\int_{Y,\alpha} \frac{x_1 dx_2 \wedge dx_3 - x_2 dx_1 \wedge dx_3 + x_3 dx_1 \wedge dx_2}{(x_1^2+x_2^2+x_3^2)^m}.$$

(b) Repeat (a) when
$$\alpha(u,v) = \left(u, v, -\sqrt{1-u^2-v^2}\right).$$

4. (Munkres §34.1) Let M be an n-manifold in $\mathbb{R}^n$. Let α, β be coordinate patches on M such that $\det D\alpha > 0$ and $\det D\beta > 0$. Show that α and β overlap positively if they overlap at all.