

Name: _____ ID: _____

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. Each problem is worth 5 points.

1. Are the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$ linearly independent? Justify your answer.

Solution. Yes, they are linearly independent, because none of them are redundant.

Indeed, $\begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$ is not a scalar multiple of $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ □

2. Consider the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and compute $\ker A$.

Solution. We find the kernel by solving the equation $A\vec{x} = \vec{0}$ using Gauss–Jordan elimination.

$$\begin{aligned} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \end{array} \right] & \begin{array}{l} \leftarrow - \\ \leftarrow + \\ \leftarrow + \end{array} \xrightarrow{-4} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right] \begin{array}{l} \leftarrow - \\ \leftarrow + \end{array} \xrightarrow{-1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \leftarrow + \\ \leftarrow - \end{array} \\ & \xrightarrow{-1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{aligned}$$

So setting $x_3 = t$, we find $x_1 = t$ and $x_2 = -2t$, so we find that all solutions to $A\vec{x} = \vec{0}$ are $\vec{x} = \begin{bmatrix} t \\ -2t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, so $\ker A = \text{span} \left(\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right)$. □