

Name: \_\_\_\_\_ ID: \_\_\_\_\_

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. Each problem is worth 5 points.

1. Suppose we have a linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ , that satisfies

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

Find the matrix  $A$  for the linear transformation  $T$ , and then compute  $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$ .

*Solution.* We have

$$A = \begin{bmatrix} | & | \\ T(\vec{e}_1) & T(\vec{e}_2) \\ | & | \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 1 & -1 \end{bmatrix},$$

and therefore

$$T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

□

2. Consider the linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by  $T(\vec{x}) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \vec{x}$ . In the empty coordinate system below (on the right), draw the vectors  $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$  and  $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$ .

*Solution.* We have

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}.$$

□

