

Print your name: _____

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. Each problem is worth 5 points.

1. Consider the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Which of the matrices are in reduced row-echelon form (rref)? No justification needed. For the matrices in reduced row-echelon form, compute their rank.

Solution. The matrices A and C are in reduced row-echelon form. The rank of A is 2 since there are two leading zeros, and the rank of C is 3 since there are three leading zeros. $\square$

2. Find the reduced row-echelon form of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$.

Solution. We perform Gauss–Jordan elimination:

$$\begin{aligned} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix} &\xrightarrow{\substack{\leftarrow -1 \\ \leftarrow + \\ \leftarrow +}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 5 \end{bmatrix} \xrightarrow{\substack{\leftarrow -2 \\ \leftarrow +}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\substack{\leftarrow -1 \\ \leftarrow + \\ \leftarrow -2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \\ &\xrightarrow{\substack{\leftarrow + \\ \leftarrow -1}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Therefore $\text{rref} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$ $\square$