MAT 203 MIDTERM II

THURSDAY NOVEMBER 13, 2025
11:00–12:20PM

Name: ______________________________ ID: ______________

Instructions.

(1) Fill in your name and Stony Brook ID number.

(2) This exam is closed-book; no electronic devices. You are only allowed to have one (1) sheet of your own notes.

(3) You have 80 minutes to complete this exam.

(4) You must justify all your answers and show all your work. Even a correct answer without any justification will result in no credit.

1. (a) (5 pts) Compute the limit $\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y}$.

Solution.

$$\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(1,1)} x+y = 2.$$

□

(b) (5 pts) The function $f(x,y) = \ln(3x^2+7y^2+1)$ is differentiable. Find the equation for the tangent plane of its graph at the point $(x,y) = (1,0)$.

Solution. We compute the partial derivatives at $(1,0)$.

$$f_x(x,y) = \frac{6x}{3x^2+7y^2+1}, \quad f_y(x,y) = \frac{14y}{3x^2+7y^2+1},$$

so $f_x(1,0) = \frac{3}{2}$ and $f_y(1,0) = 0$. Therefore the equation for the tangent plane at $(x,y) = (1,0)$ is given by

$$f_x(1,0)(x-1) - (z - f(1,0)) = 0 \Leftrightarrow \frac{3}{2}(x-1) - (z - \ln(4)) = 0.$$

□

2. Consider the function $f(x, y) = 2xy - x$.

(a) (2 pts) Find all critical points for $f(x, y)$.

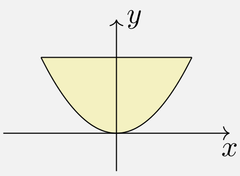
Solution. We first find the gradient:

$$\nabla f(x, y) = \langle 2y - 1, 2x \rangle.$$

The critical points are those point (x, y) such that $\nabla f(x, y) = \vec{0}$. So $\langle 2y - 1, 2x \rangle = \langle 0, 0 \rangle$ gives the only critical point $(x, y) = \left(0, \frac{1}{2}\right)$. □

(b) (8 pts) Find the global maximum, and global minimum of $f(x, y)$ in the region of the xy-plane bounded by the line $y = 1$, and the parabola $y = x^2$.

Solution. We draw the region in the xy-plane:



First, from part (a), the only critical point is given by $(x, y) = \left(0, \frac{1}{2}\right)$. This point belongs to the region above, and the value of the function at that point is $f\left(0, \frac{1}{2}\right) = 0$.

Next, we consider the boundary of the region. It consists of two parts: The first one is the straight line $y = 1$ between $-1 \le x \le 1$, and the other is $y = x^2$ for $-1 \le x \le 1$. Along the first part of the boundary, the function is given by

$$f_1(x) = 2x - x = x.$$

This function has no critical points in the interval $-1 \le x \le 1$, so we check the endpoints of the interval: $(x, y) = (-1, 1)$ and $(x, y) = (1, 1)$. The values at these points are

$$f(-1, 1) = -2 - (-1) = -1 \quad \text{and} \quad f(1, 1) = 2 - 1 = 1.$$

For the second boundary segment, we have $y = x^2$ for $-1 \le x \le 1$. The function along this boundary segment is

$$f_2(x) = 2x^3 - x.$$

We find the critical points of this function. First $f_2'(x) = 6x^2 - 1 = 0$ gives $x = \pm\frac{1}{\sqrt{6}}$. Each of these points belong to the interval $-1 \le x \le 1$. The values at these critical points are

$$f_2\left(\frac{1}{\sqrt{6}}\right) = \frac{1}{3\sqrt{6}} - \frac{1}{\sqrt{6}}, \quad f_2\left(-\frac{1}{\sqrt{6}}\right) = -\frac{1}{3\sqrt{6}} + \frac{1}{\sqrt{6}}.$$

The potential values for global max/min of the function f are therefore 0, -1, 1, $\frac{1}{3\sqrt{6}} - \frac{1}{\sqrt{6}}$ and $-\frac{1}{3\sqrt{6}} + \frac{1}{\sqrt{6}}$. The global max is 1, and the global min is -1. □

3. (10 pts) Let B be the rectangular box

$$B = [\pi, 2\pi] \times [0, \pi] \times [0, 1] = \left\{(x, y, z) \in \mathbb{R}^3 \mid \pi \leq x \leq 2\pi,\ 0 \leq x \leq \pi,\ 0 \leq z \leq 1\right\}.$$

Compute the following triple integral

$$\iiint_B e^z \cos(x + y)\, dxdydz.$$

Solution. Since we integrate over a rectangular box, the integral is an iterated integral.

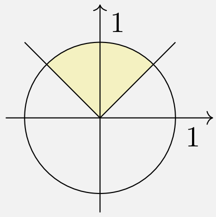
$$\begin{aligned}
\iiint_B e^z \cos(x+y)\, dxdydz &= \int_\pi^{2\pi} \int_0^\pi \int_0^1 e^z \cos(x+y)\, dzdxdy \\
&= \int_\pi^{2\pi} \int_0^\pi [e^z]_0^1 \cos(x+y)\, dydx \\
&= (e-1) \int_\pi^{2\pi} \int_0^\pi \cos(x+y)\, dydx \\
&= (e-1) \int_\pi^{2\pi} [\sin(x+y)]_{y=0}^{y=\pi}\, dx \\
&= (e-1) \int_\pi^{2\pi} \sin(x+\pi) - \sin(x)\, dx \\
&= (e-1)[-\cos(x+\pi) + \cos(x)]_\pi^{2\pi} \\
&= (e-1)[(-\cos(3\pi) + \cos(2\pi)) + (\cos(2\pi) - \cos(\pi))] \\
&= (e-1)[(-(-1)+1) + (1-(-1))] = 4(e-1)
\end{aligned}$$

□

4. (10 pts) Let D be the domain in the xy-plane consisting of those points (x, y) satisfying the inequalities $y \geq |x|$ and $x^2 + y^2 \leq 1$. Compute the double integral

$$\iint_D y\, dxdy.$$

Solution. We draw the region D:



In polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, the region D is described as the points (r, θ) such that $0 \leq r \leq 1$ and $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$. Therefore

$$\iint_D y\, dxdy = \int_0^1 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} r\sin\theta\, r drd\theta = \left(\int_0^1 r^2\, dr\right)\left(\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin\theta\, d\theta\right)$$

$$= \left[\frac{r^3}{3}\right]_0^1 [-\cos\theta]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \frac{1}{3}\cdot\left(-\cos\frac{3\pi}{4} + \cos\frac{\pi}{4}\right) = \frac{\sqrt{2}}{3}$$

□