MAT 203 L5 Tue Sep 9

Recall:

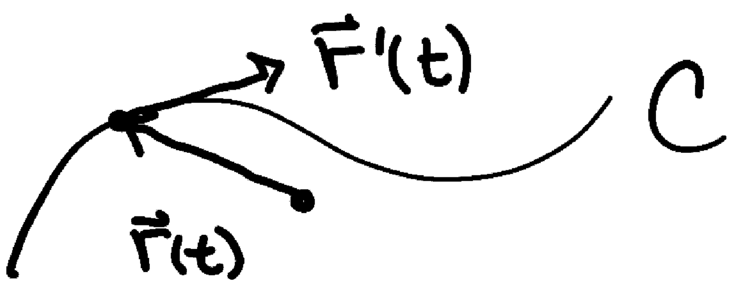
- $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

Parametrization of a curve in space.

- The tangent vector at time t is $\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$

In the plane: $\vec{r}\,'(t) = \langle x'(t), y'(t) \rangle$

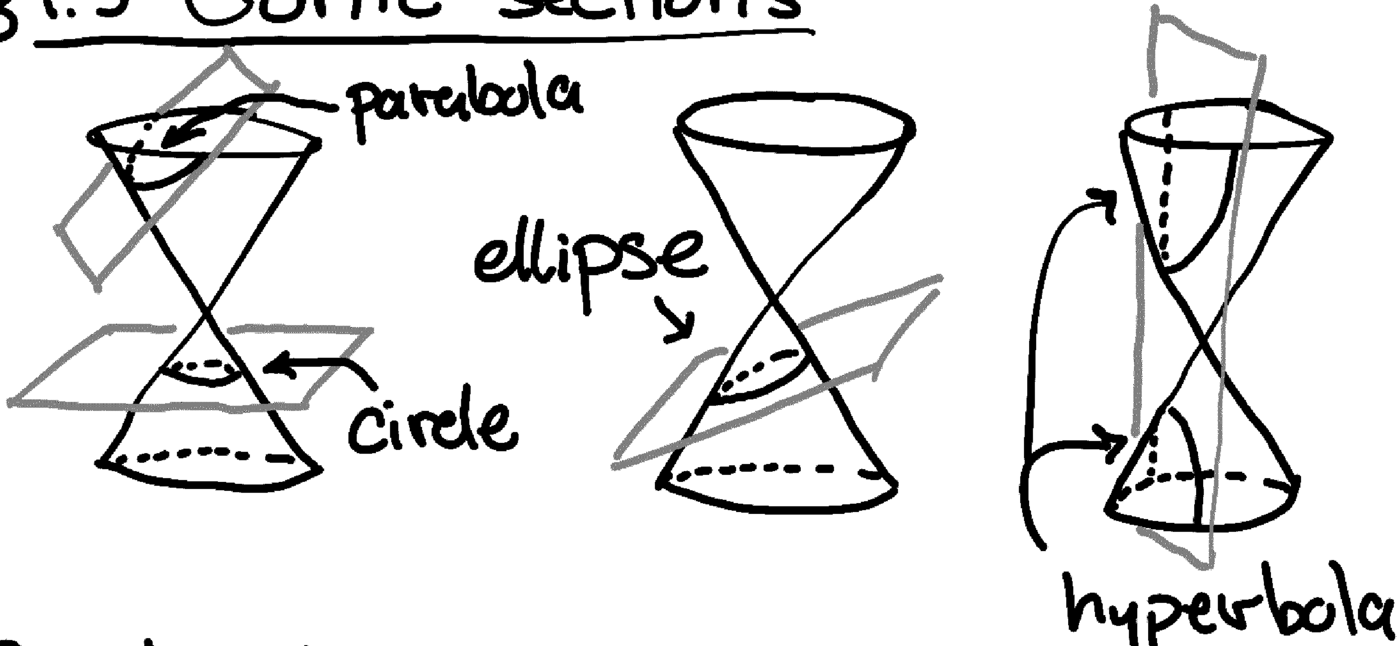
Slope $= \dfrac{y'(t)}{x'(t)}$ (if $x'(t) \neq 0$)



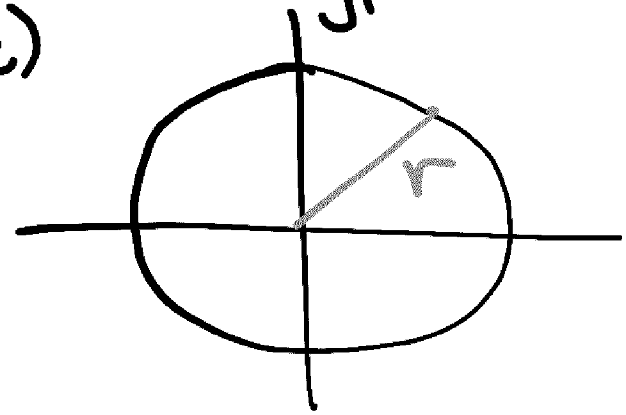
- Arc length of curve between $a \le t \le b$

$$L = \int_a^b \|\vec{r}\,'(t)\|\, dt.$$

§1.5 Conic sections



Circle (in the plane)



$$\boxed{x^2+y^2=r^2}$$

r = radius

center = $(0,0)$

A circle centered at (x_0, y_0) is described by

$$(x-x_0)^2+(y-y_0)^2=r^2$$

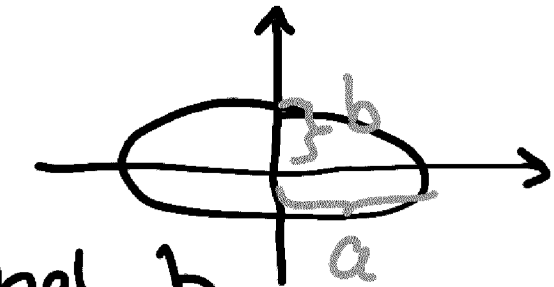
Parametrization: $\begin{cases} x = x_0 + r\cos\theta \\ y = y_0 + r\sin\theta \end{cases}$

$$\vec{r}(\theta) = \langle x_0 + r\cos\theta,\ y_0 + r\sin\theta \rangle$$

Ellipse:

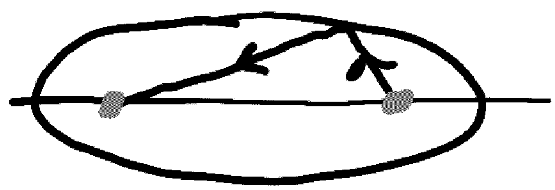
$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

Center: (x_0, y_0)



Two radii: a and b.

Foci: If $a>b$: $(x_0 \pm \sqrt{a^2-b^2}, y_0)$



If $b>a$:

$(x_0, y_0 \pm \sqrt{b^2-a^2})$

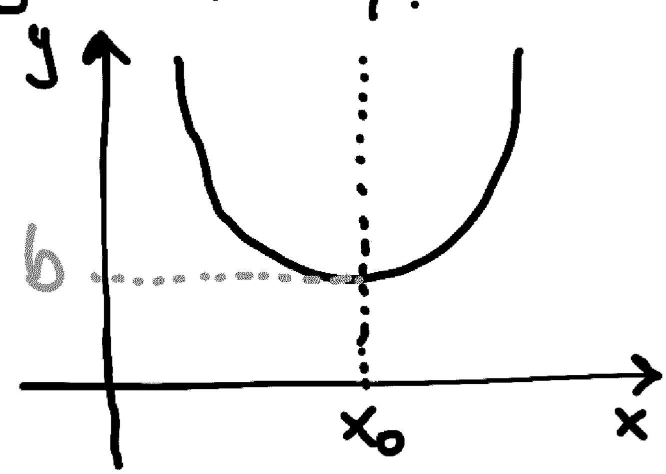
Parametrization:

$$\begin{cases} x = x_0 + a\cos\theta \\ y = x_0 + b\sin\theta \end{cases}$$

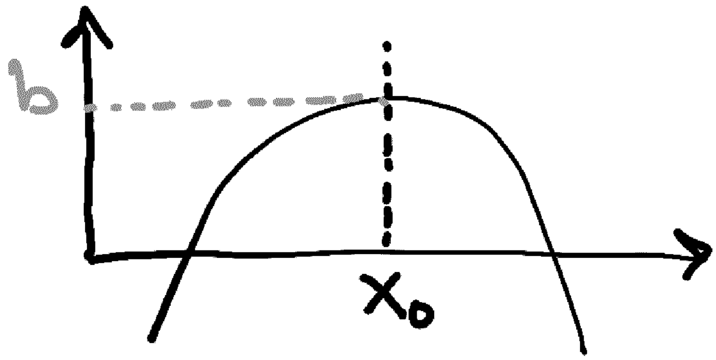
$$\vec{r}(\theta) = \langle x_0 + a\cos\theta,\ y_0 + b\sin\theta \rangle.$$

Parabola:

$$y = a(x-x_0)^2 + b.$$



$\boxed{a>0}$



$\boxed{a<0}$

Parametrization:

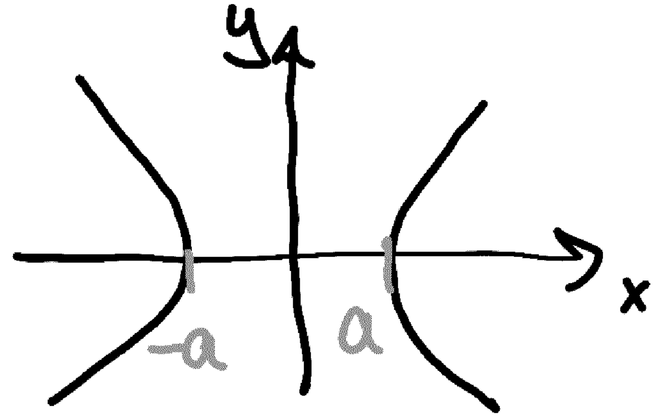
$$\begin{cases} x = t \\ y = a(t-x_0)^2 + b \end{cases}$$

$$\vec{r}(t) = \langle t, a(t-x_0)^2 + b \rangle$$

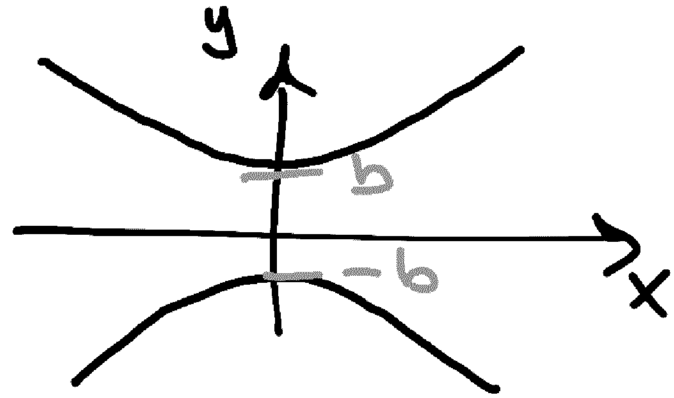
Hyperbola:

$$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = \pm 1$$

If RHS $= +1$:



If RHS $= -1$:



Parametrization:

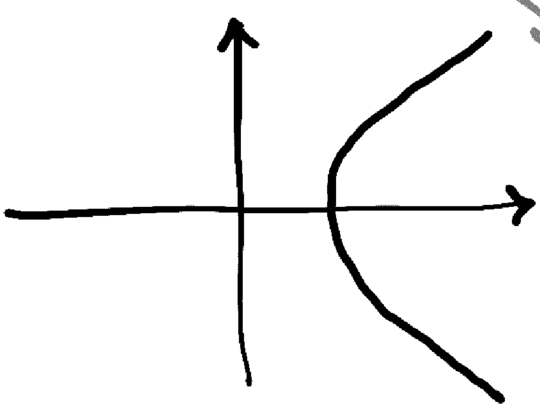
$$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$$

$x = t$, then we solve for y:

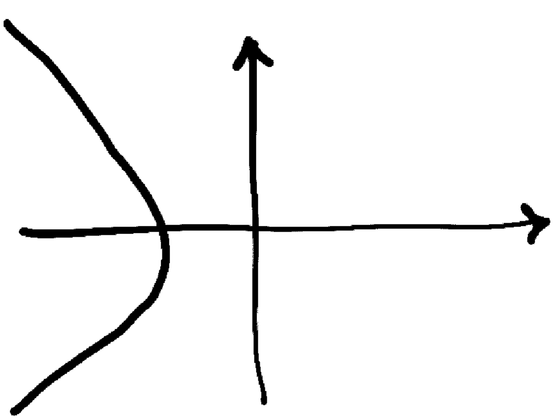
$$\frac{(t-x_0)^2}{a^2} = 1 + \frac{(y-y_0)^2}{b^2}$$

$$\frac{b^2(t-x_0)^2}{a^2} - b^2 = (y-y_0)^2$$

$$y = y_0 \pm \sqrt{\frac{b^2(t-x_0)^2}{a^2} - b^2}$$



- the $+$ sign will parametrize the <u>right side</u> of this hyperbola.



- the $-$ sign will parametrize the <u>left side</u> of this hyperbola.

§2.6 Quadratic surfaces

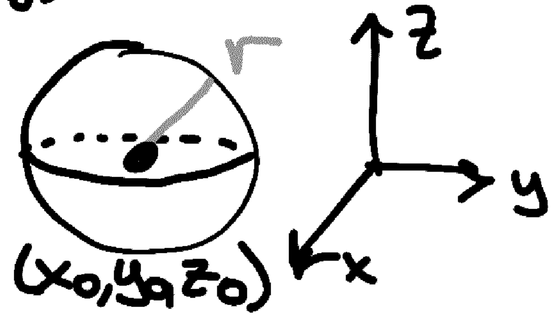
Surfaces in space that can be described by:

$$\boxed{Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Jz+K=0}$$

Sphere: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$

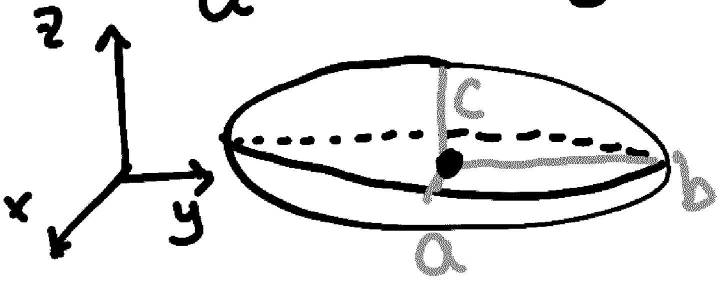
radius $= r$

center $= (x_0,y_0,z_0)$



Ellipsoid:

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$$

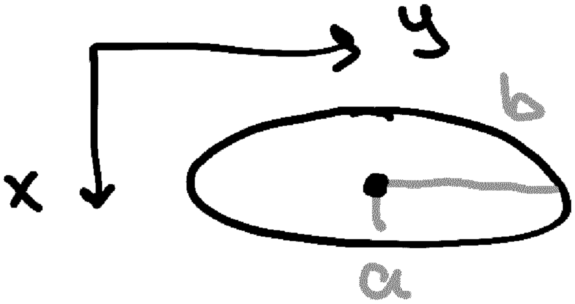


radii: a, b, c

center $= (x_0,y_0,z_0)$

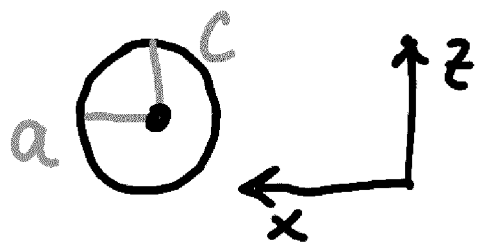
In the xy-plane: (when $z=z_0$)

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1 \quad \text{ellipse}$$



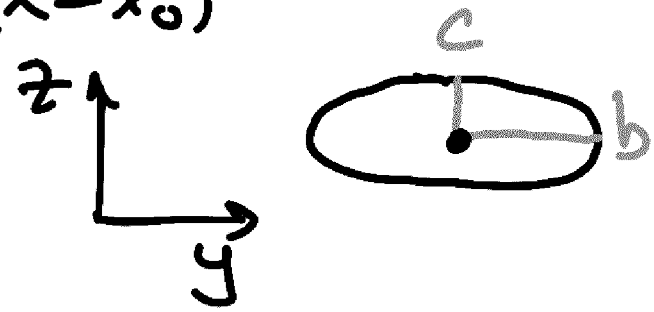
In the xz-plane: (when $y = y_0$)

$$\frac{(x-x_0)^2}{a^2} + \frac{(z-z_0)^2}{c^2} = 1$$



In the yz-plane: ($x = x_0$)

$$\frac{(y-y_0)^2}{b^2} + \frac{(z-z_0)^2}{c^2} = 1$$

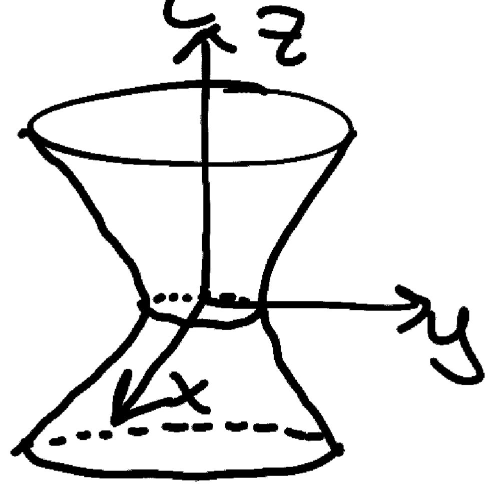


Hyperboloid:

note 1 minus sign

$$+\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} - \frac{(z-z_0)^2}{c^2} = 1$$

"one-sheeted"



xy-plane:

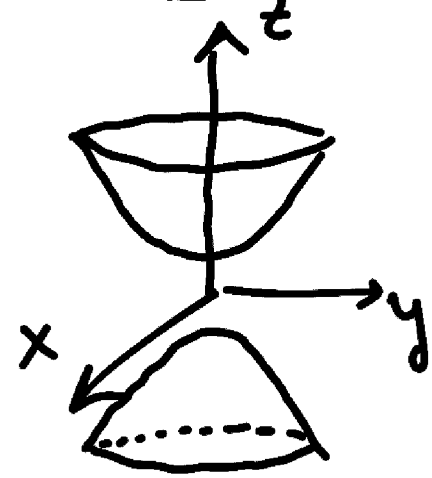
ellipse

yz- and xz-planes: hyperbolas

"two-sheeted hyperboloid"

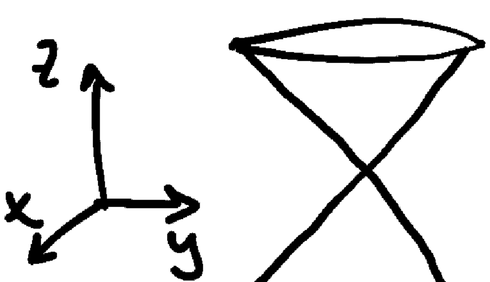
$$-\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$$

two minus signs



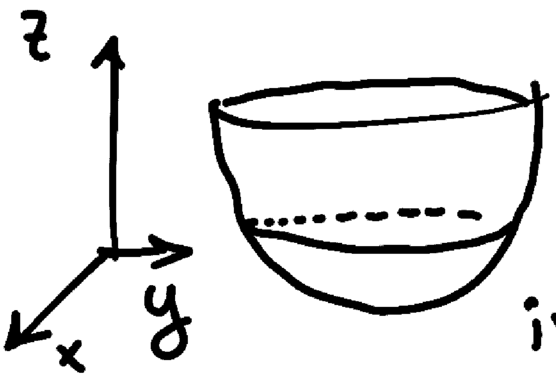
- # minus signs (1 or 2) tells us how many sheets the hyperboloid has.

Cone: $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=\frac{(z-z_0)^2}{c^2}$



Elliptic paraboloid:

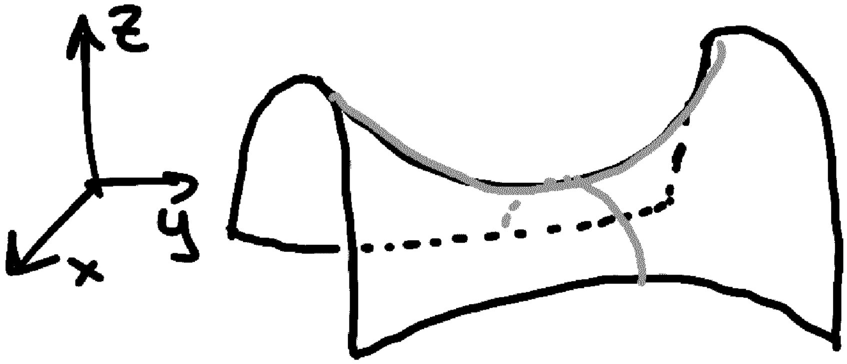
$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=z$$



In the xy-slice $z=p$ an ellipse, and in yz- and xz-slices (meaning $x=q$ and $y=r$) its a parabola.

<u>Hyperbolic paraboloid</u>

$$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=z$$



"Saddle"

In xy-slice $z=p$: hyperbola

In xz- and yz-slices $x=q$ and $y=r$: parabolas.

<u>Ex</u>: Identify the quadratic surface $9x^2+y^2-z^2+2z-10=0$.

<u>Sol</u>: Complete the squares:

$$9x^2+y^2-(z^2-2z+10)=9x^2+y^2-((z-1)^2+9)=0$$

$$\Leftrightarrow 9x^2 + y^2 - (z-1)^2 = 9$$

$$\Leftrightarrow x^2 + \frac{y^2}{9} - \frac{(z-1)^2}{9} = 1$$

One minus sign, so it's a one-sheeted hyperboloid.
