

MAT 203

L1

Tue Aug 26

Admin stuff: (see syllabus)

Me: Johan Asplund Math 3-116

Office hours: TuTh 12:30-1:30 PM

Course web page: M 6-7 PM (MLC)

math.Stonybrook.edu/~jasplund/mat203_fall25

Textbook: Openstax Calculus Volume 3
(free; available online)

Weekly homeworks/quizzes:

- Weekly pool of 6-8 problems,
- 2 are homework problems to be handed in at recitation (due next week)
- Quiz last 15 min of each recitation where you will be asked to solve 1 (randomly chosen) problem from the pool.

First quiz: Next week.

Important dates:

- Midterm 1: Tu Sep 25
- Midterm 2: Th Nov 13
- Final exam: W Dec 10

2.2.88
2.4.190

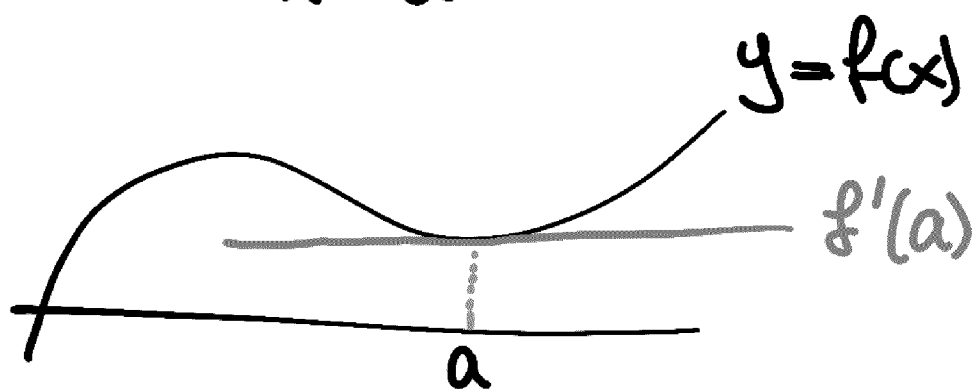
2.1.6
2.1.10
2.2.92
2.3.126
2.4.186

§ Ideas/goals of MAT 203

Calculus I:

$$f(x) = x^3 - x$$

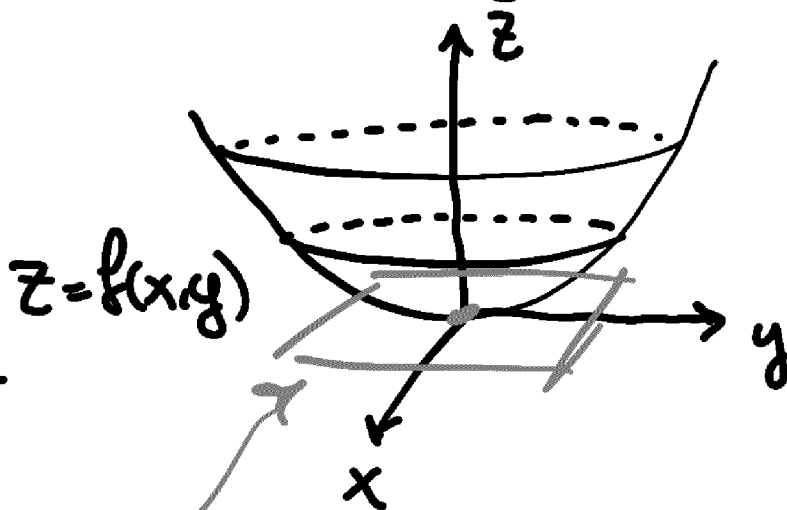
$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$



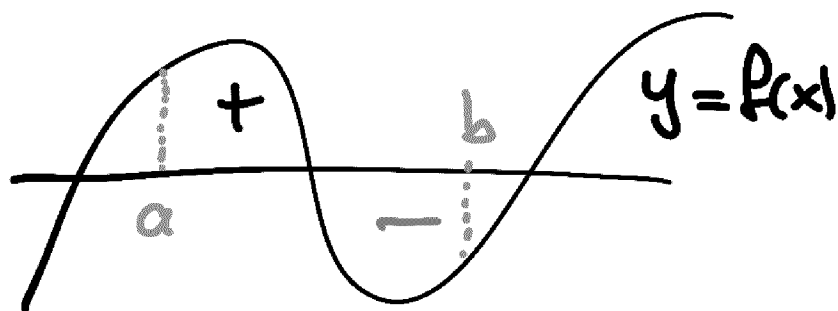
What if $f(x,y) = x^3 - x + y^2 - y$?
What is the rate of change
as x changes? as y changes?
or both?

$$f(x,y) = x^2 + y^2$$

We do not
have a tangent
line, but a
tangent plane.



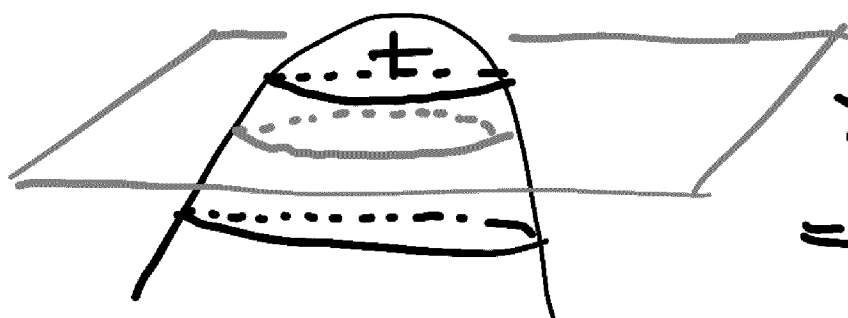
Calculus II :



$$\int_a^b f(x) dx = \text{Signed area under the graph } y = f(x).$$

= Summing up values of $f(x)$ over the interval $[a, b]$.

For functions of multiple variables one computes a signed volume:



$$f(x, y) = 1 - x^2 - y^2.$$

$$V = \iint_{x^2 + y^2 \leq 1} f(x, y) dx dy$$

There are at least 4 other kinds of integrals we will learn in this class.

Fund thm of calculus:

$F(x) = \int_a^x f(t) dt$, then $F(x)$ is differentiable & $F'(x) = f(x)$.

There are different versions of this for functions of multiple variables:

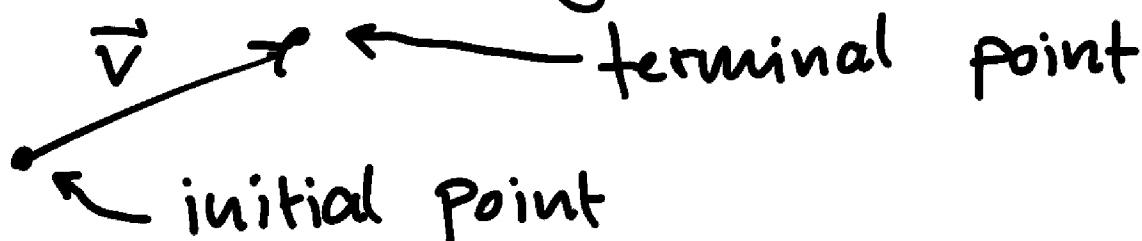
- Green's theorem
- Stokes' theorem
- Gauss' theorem
(divergence theorem).

- Important in applications such as physics (e.g. Maxwell's equations).

§2.1 Vectors in the plane & space

2.2

Def: A vector is a quantity that has both a magnitude & direction.



We write an arrow over v to indicate that v is a vector.

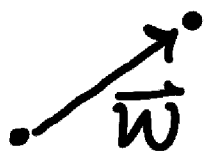
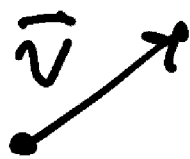
In print, boldface fonts are sometimes used, but it's hard to use in handwriting.

Magnitude of $\vec{v}$ is denoted by $\|\vec{v}\|$.

- A vector from the point P to the point Q is denoted $\overrightarrow{PQ}$.



- Two vectors are "equivalent" (or the same) if they have the same magnitude & the same direction.



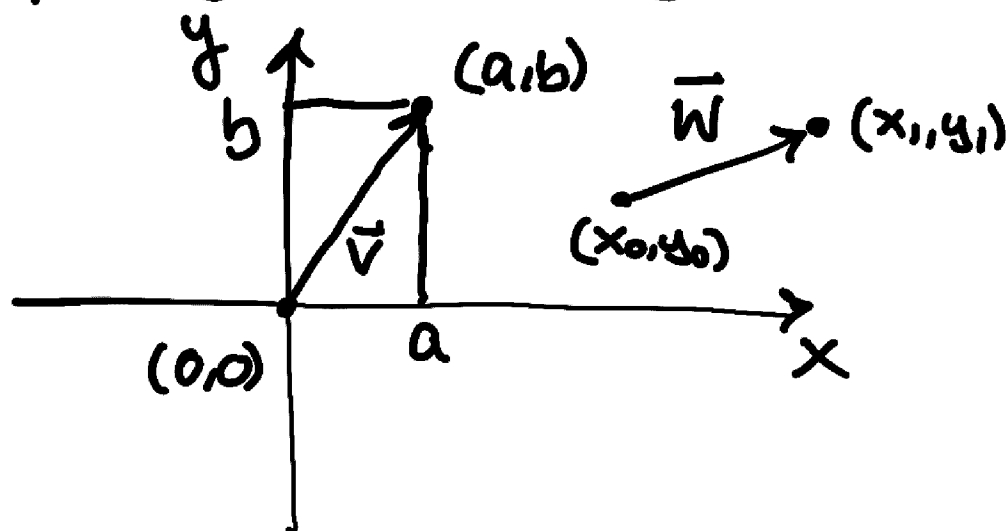
$$\vec{v} = \vec{w}.$$

Even if their initial and terminal points are different, their magnitudes & directions are the same.

A vector $\vec{v}$ with $\|\vec{v}\| = 0$ is called the zero vector. It has no direction. (It's the only vector without one.)

Cartesian plane:

$$\mathbb{R}^2 = \{ (x, y) \mid x, y \in \mathbb{R} \}$$



$\vec{v}$ starts at $(0,0)$ & ends at (a,b) .

it's denoted by $\vec{v} = \langle a, b \rangle$, where a and b are the components of $\vec{v}$.

The vector $\vec{w}$ is equivalent to the vector starting at $(0,0)$ and ending at $(x_1 - x_0, y_1 - y_0)$, so

$$\vec{w} = \langle x_1 - x_0, y_1 - y_0 \rangle.$$

Zero vector: $\vec{0} = \langle 0, 0 \rangle$.

Magnitude of $\langle a, b \rangle$ is

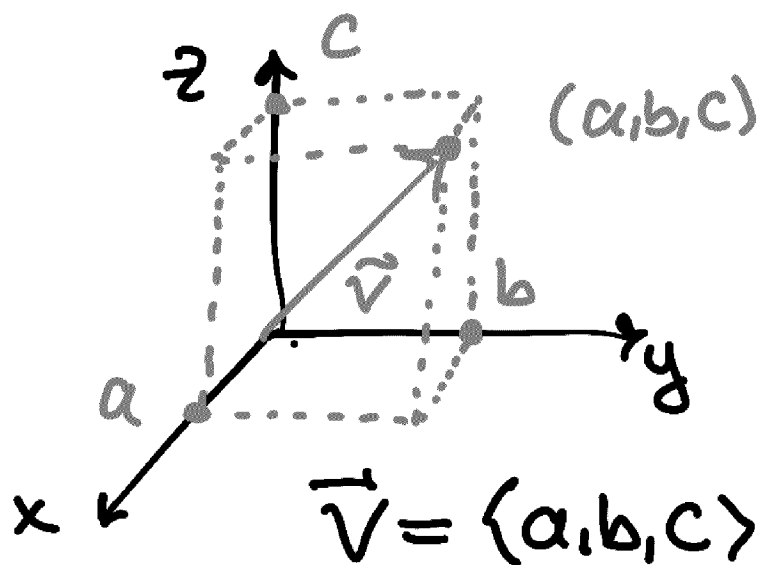
$$\| \langle a, b \rangle \| = \sqrt{a^2 + b^2}$$

Magnitude of vector from (x_0, y_0) to (x_1, y_1) is

$$\| \langle x_1 - x_0, y_1 - y_0 \rangle \| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

Cartesian space:

$$\mathbb{R}^3 = \{ (x, y, z) \mid x, y, z \in \mathbb{R} \}$$



$\vec{V}$ starts at $(0,0,0)$ and ends at (a,b,c) .

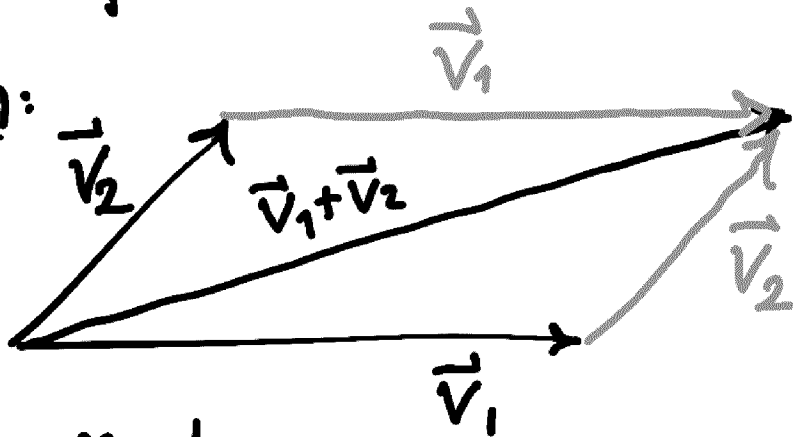
$$\|\vec{V}\| = \sqrt{a^2 + b^2 + c^2}.$$

If $\vec{w}$ starts at (x_0, y_0, z_0) and ends at (x_1, y_1, z_1) , then

$$\|\vec{w}\| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}.$$

Vector operations:

Addition:



In components:

$\vec{v}_1 = \langle x_1, y_1, z_1 \rangle$, $\vec{v}_2 = \langle x_2, y_2, z_2 \rangle$ then

$$\vec{v}_1 + \vec{v}_2 = \langle x_1 + x_2, y_1 + y_2, z_1 + z_2 \rangle$$

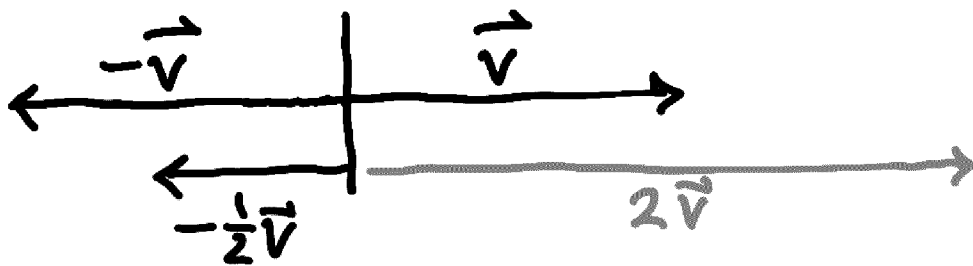
note: $\|\vec{v}_1 + \vec{v}_2\| \leq \|\vec{v}_1\| + \|\vec{v}_2\|$.

(triangle inequality)

Scalar multiplication:

If $\vec{v}$ is a vector, and r is a scalar (real number)

The vector $r\vec{v}$ is the vector in the same direction (if $r > 0$), or opposite direction (if $r < 0$) as $\vec{v}$, but w/ length $|r| \|\vec{v}\|$.



If $\vec{v} = \langle x, y, z \rangle$, then

$r\vec{v} = \langle rx, ry, rz \rangle$. Note $0 \cdot \vec{v} = \vec{0}$ for any vector $\vec{v}$.

• Now we can subtract vectors:

$$\vec{v}_1 - \vec{v}_2 = \vec{v}_1 + (-1)\vec{v}_2.$$

$$= \langle x_1, y_1, z_1 \rangle + (-1)\langle x_2, y_2, z_2 \rangle = \langle x_1 - x_2, y_1 - y_2, z_1 - z_2 \rangle$$

• A unit vector is a vector of length 1. Given a vector $\vec{v} \neq \vec{0}$, its length is $\|\vec{v}\|$. Therefore

$\frac{1}{\|\vec{v}\|} \vec{v}$ has the same direction as $\vec{v}$, but $\|\frac{1}{\|\vec{v}\|} \vec{v}\| = \frac{1}{\|\vec{v}\|} \cdot \|\vec{v}\| = 1$

So $\frac{1}{\|\vec{v}\|} \vec{v}$ is a unit vector.

$\vec{v} = \langle x, y, z \rangle$, $\|\vec{v}\| = \sqrt{x^2 + y^2 + z^2}$, then

$$\frac{1}{\|\vec{v}\|} \vec{v} = \left\langle \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}} \right\rangle$$

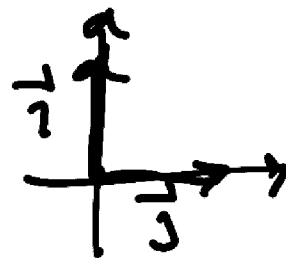
Properties of vector operations:

$\vec{u}, \vec{v}, \vec{w}$ vectors, r, s scalars

- (1) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- (2) $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
- (3) $\vec{u} + \vec{0} = \vec{u}$
- (4) $\vec{u} + (-\vec{u}) = \vec{0}$
- (5) $r(s\vec{u}) = (rs)\vec{u}$
- (6) $(r+s)\vec{u} = r\vec{u} + s\vec{u}$
- (7) $r(\vec{u} + \vec{v}) = r\vec{u} + r\vec{v}$
- (8) $1\vec{u} = \vec{u}, 0\vec{u} = \vec{0}$
- (9) $\|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$ (triangle inequality)

Standard unit vectors

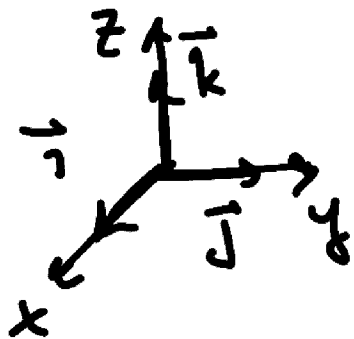
Plane: $\vec{i} = \langle 1, 0 \rangle$
 $\vec{j} = \langle 0, 1 \rangle$



$$\vec{v} = \langle x, y \rangle = x\vec{i} + y\vec{j}$$

Space: $\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$
 $\vec{k} = \langle 0, 0, 1 \rangle$

$$\vec{V} = \langle x, y, z \rangle = x\vec{i} + y\vec{j} + z\vec{k}.$$



Ex. $P = (0, 0, 1)$, $Q = (1, 0, 1)$, $R = (1, 2, 3)$
 $S = (0, 1, 0)$.

Let $\vec{u} = \overrightarrow{PQ}$, $\vec{v} = \overrightarrow{RS}$

$$\vec{u} = \langle 1-0, 0-0, 1-1 \rangle = \langle 1, 0, 0 \rangle$$

$$\vec{v} = \langle 0-1, 1-2, 0-3 \rangle = \langle -1, -1, -3 \rangle$$

$$\vec{u} + \vec{v} = \langle 1-1, 0-1, 0-3 \rangle = \langle 0, -1, -3 \rangle$$

$$\|\vec{u} + \vec{v}\| = \sqrt{0^2 + (-1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}.$$
