MAT 127

COVERAGE OF MIDTERM I

The midterm will consist of 5 problems, each one worth 20 points each. It will include problems on topics on **sequences**, **infinite series** and **power series**. Taylor series will *not* be covered on the first midterm.

Below is a more detailed list of learning goals for each section. Keep in mind that because the midterm only consists of 5 problems, it is impossible to test each and every one of the learning goals listed below. All section references are to Stewart.

1. Learning goals

Sequences: (All of §8.1.)

Learning goals:

- Understand the notion of a sequence and work with different ways of describing sequences (formulas for the general term, recursive formulas, relation to functions, plots of sequences), compute several terms of a sequence given as above.
- Understand intuitively the notions of convergence and limit of a sequence.
- Identify and correctly use terminology describing properties of sequences (increasing, decreasing, monotonic, bounded above, bounded below).

Infinite series: (All of §8.2, §8.3: Integral test, p-series and comparison test only, §8.4: Alternating series test and ratio test only.)

Learning goals:

- Understand the notion of series; distinguish between sequences and series.
- Know the definition of an infinite series as the limit of the partial sum sequence.
- Recognize the geometric series and know how to identify whether it converges or diverges; in case of convergence being able to compute the sum.
- Recognize the harmonic series and know that it diverges.
- Recognize and being able to carry out simple examples of telescoping series.
- Know the statement of the divergence test, ratio test, integral test, comparison test and the alternating series test and how to use them.
- Recognize a p-series and knowing for which values it converges/diverges.

Power series: (All of §8.5 and §8.6.)

Learning goals:

- Identify a power series.
- Being able to find the radius and interval of convergence using the ratio test, including endpoints of the convergence interval.
- Differentiate and integrate power series to find new power series from old ones. Find radius and interval of convergence of the resulting series.

2. What convergence test to use?

Being able to know **which** convergence test to use is an important skill to succeed on the first midterm. This skill is best learned by practicing; do a lot of problems. Eventually you begin to get a sense of what convergence test is likely to work in a specific situation. (This is similar to the question "How do I know which technique to use when integrating a function?" that you likely have asked and answered in the past.) Below is one possible flow chart that is usually similar to how I personally deal with this problem when faced with a new numerical series:

- Try the divergence test: Do the terms go to zero as $n \to \infty$? If not, it diverges by the divergence test.

- Is it a p-series, i.e. of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$? If so, it converges for $p > 1$, and diverges otherwise.
- Is it a geometric series, i.e. of the form $\sum_{n=0}^{\infty} ar^n$? If so, it converges for $|r| < 1$. The sum is $\frac{a}{1-r}$, assuming that the index starts at 0.
- Is the series alternating? Try the alternating series test.
- Try the ratio test. Compute $\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$. If $\rho > 1$, it diverges, if $\rho < 1$ converges and if $\rho = 1$ the ratio test is inconclusive.
- Does the series "look like" another series that you know? For example $\sum_{n=1}^{n} \frac{1}{n^2+1}$ "looks like" $\sum_{n=1}^{n} \frac{1}{n^2}$ which we recognize as a p-series with $p = 2$. Using the comparison test is likely a good idea. Remember that the terms need to be positive in order for this to be applicable.
- Is the n-th term a_n described by a function f that looks like something you can integrate? Try to use the integral test.
- Last resort: Compute the partial sums $S_N = \sum_{n=1}^{N} a_n$ and take the limit as $N \to \infty$. This will be easy in the case of a telescoping series such as $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$.