

MAT 127 MIDTERM I

PRACTICE PROBLEMS

1. Determine whether the following sequences converge or diverge.
(a) (10 pts)

$$a_n = \frac{2n^3 + n - 1}{4n^3 + n^2 + n - 1}$$

- (b) (10 pts)

$$a_n = \frac{2^n + \sin(n)}{n!}$$

2. Determine whether the following series converge or diverge.

(a) (10 pts)

$$\sum_{n=1}^{\infty} \left(\frac{1}{n^2} + \frac{7^n}{6^{n+1}} \right)$$

(b) (10 pts)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$$

3. (20 pts) Is the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ convergent or divergent? [*Hint: Use the integral test and note the starting point of the series.*]

4. Find the radius and interval of convergence of the following power series.

(a) (10 pts)

$$\sum_{n=1}^{\infty} \frac{(-1)^n (3x)^n}{n!}$$

(b) (10 pts)

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$$

5. Find a power series representation of the following functions. Also determine the interval of convergence.

(a) (10 pts) $f(x) = \frac{2}{3+x^2}$

(b) (10 pts) $g(x) = \frac{2}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right)$ [*Hint: What is the derivative of $g(x)$?*]