Print your name: _____

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. Each problem is worth 5 points.

1. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^3}$. Remember to check the endpoints of the interval.

Ratio test: $S = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(x-2)^{n+1}/(n+1)^3}{(x-2)^n/n^3}\right|$

$= \lim_{n\to\infty} |x-2| \cdot \frac{n^3}{(n+1)^3} = |x-2| \underbrace{\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^3}_{=1} = |x-2|$

Conv if $|x-2|<1 \iff -1<x-2<1 \iff 1<x<3$

div if $|x-2|>1 \iff x<1$ or $x>3$

$x=1$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ conv by alt. series test

$x=3$ $\sum_{n=1}^{\infty} \frac{1}{n^3}$ conv (p-series w/ $p=3>1$)

Interval of conv $= 1 \le x \le 3$, radius $= 1$

2. The power series $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ has radius of convergence $R = \infty$. Calculate $f'(x)$. (Extra question worth no credits: Based on this calculation, can you guess which function the power series $f(x)$ represents?)

$f'(x) = \frac{d}{dx} \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{\frac{d}{dx} x^n}{n!} = \sum_{n=1}^{\infty} \frac{n x^{n-1}}{n!}$

$= \sum_{n=1}^{\infty} \frac{n x^{n-1}}{n\cdot(n-1)!} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!} = \sum_{n=0}^{\infty} \frac{x^n}{n!} = f(x)$

Extra Q:

Solution to $f'(x) = f(x)$ is $f(x) = e^x$, so must have

$$\boxed{e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}}$$