MAT 127

COVERAGE OF MIDTERM II

The midterm will consist of 5 problems, each one worth 20 points each. It will include problems on topics on **Taylor and Maclaurin series**, **slope fields and Euler's method**, **separable ODEs** and **modeling using ODEs**. Absolute and conditional convergence will *not* be covered on the second midterm (but it will be covered on the final).

Below is a more detailed list of learning goals for each section. Keep in mind that because the midterm only consists of 5 problems, it is impossible to test each and every one of the learning goals listed below. All section references are to Stewart.

1. Learning goals

Taylor and Maclaurin series: (All of §8.7.) Learning goals:

- State the general form of Taylor and Maclaurin series and polynomials.
- Find Taylor series and Taylor polynomials with a given center, by three methods: (1) computing derivatives and using the Taylor formula, (2) using substitution and arithmetic operations on known basic Maclaurin series such as $\sin x$, $\cos x$, e^x, $\frac{1}{1-x}$, $\log(1+x)$, or (3) using differentiation or integration of known series.
- **Note: You will *not* need to memorize common Taylor series: A reference page will be provided on the second midterm.**

Differential equations, slope fields and Euler's method: (All of §7.1 and §7.2.)
Learning goals:

- Explain what a differential equation is and find the order of the given differential equation.
- Understand the notion of a solution of a differential equation; check whether a given function is a solution to the given differential equation or to the initial value problem.
- Understand slope fields as a geometric interpretation of a first order ODE and use them to derive properties solutions (plot a solution curve for a given initial value problem; determine whether the solution to an initial value problem is increasing or decreasing; determine asymptotic behavior of a solution of the initial value problem).
- Being able to sketch a slope field.
- Understand and apply Euler's method to find approximations of solutions; provide a geometric interpretation of the method.

Separable ODEs and modeling: (All of §7.3 and §7.4.)
Learning goals:

- Identify separable ODEs; apply method of separation of variables to solve differential equations; solve initial-value problems.
- Compose differential equations and initial value problems to model processes described by word problems; solve the resulting problems and give a real-life interpretation of the solution; compare the behavior of solutions to qualitative predictions obtained by vector fields; verify whether the solutions make sense in the context of the problem
- Work with exponential models for growth and decay, recognize the corresponding differential equations.

2. Reference page that will be provided at the midterm

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots \quad |x| < 1$$

$$\log(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \quad x \in \mathbb{R}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \quad x \in \mathbb{R}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \quad x \in \mathbb{R}$$

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \quad |x| < 1$$

The **Taylor series of $f(x)$ centered at $x = a$** is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$