Print your name: ____________________

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits.

1. Mark whether each statement is true or false. No justification needed. (3 pts)
(a) A convergent series is absolutely convergent. **False**
(b) A series can be convergent while not being absolutely convergent. **True**
(c) An absolutely convergent series is convergent. **True**

2. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is absolutely convergent, conditionally convergent, or divergent. (5 pts)

Solution. It is not absolutely convergent, because $\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{\sqrt{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ is divergent. This is because it is a p-series with $p = \frac{1}{2} < 1$.
It is convergent, by the alternating series test. Namely the sequence $a_n = \left\{\frac{1}{\sqrt{n}}\right\}$ is decreasing because

$$a_{n+1} = \frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}} = a_n,$$

and $\lim_{n\to\infty}\frac{1}{\sqrt{n}} = 0$. Therefore the alternating series test gives that $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ is convergent. The conclusion is that $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ is conditionally convergent. □

3. Determine whether the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n n!$ is absolutely convergent, conditionally convergent, or divergent. (3 pts)

Solution. Since $\lim_{n\to\infty}(-1)^n n! \neq 0$, the divergence test tells us that it is divergent. Since the series is not convergent it can not be absolutely convergent. □