Print your name: ______________________________

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. Each problem is worth 5 points.

For reference:

$$\cos x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

1. Find the Maclaurin series of $f(x) = x^2 e^{x^2}$.

Solution. From the reference we replace x with x^2 in the infinite series, and multiply it by x^2 to obtain

$$x^2 e^{x^2} = x^2 \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n+2}}{n!} = x^2 + x^4 + \frac{x^6}{2!} + \frac{x^8}{3!} + \cdots$$

□

2. Compute the limit $\lim\limits_{x\to 0} \dfrac{\cos(3x) - 1}{x^2}$

Solution. From the reference we have $\cos(3x) = \sum_{n=0}^{\infty}(-1)^n \frac{(3x)^{2n}}{(2n)!} = 1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} - \cdots$. We substitute this into the limit and simplify:

$$\begin{aligned}\lim_{x\to 0} \frac{\cos(3x) - 1}{x^2} &= \lim_{x\to 0} \frac{\left(1 - \frac{9x^2}{2!} + \frac{81x^4}{4!} - \cdots\right) - 1}{x^2} = \lim_{x\to 0} \frac{-\frac{9x^2}{2!} + \frac{81x^4}{4!} - \cdots}{x^2} \\ &= \lim_{x\to 0} -\frac{9}{2} + \underbrace{\frac{81x^2}{4!} - \cdots}_{\to 0} = -\frac{9}{2}.\end{aligned}$$

□