Print your name: ____________________

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. Each problem is worth 5 points.

1. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-6)^n}{n!}(x-10)^n$. Remember to check the endpoints of the interval.

Solution. We use the ratio test.

$$\rho = \lim_{n\to\infty} \left| \frac{\frac{(-6)^{n+1}}{(n+1)!}(x-10)^{n+1}}{\frac{(-6)^n}{n!}(x-10)^n} \right| = \lim_{n\to\infty} \left| (-6) \cdot \frac{n!}{(n+1)!} \cdot (x-10) \right|$$
$$= \lim_{n\to\infty} \frac{6|x-10|}{n+1} = 0,$$

therefore $\rho = 0 < 1$ for all x. It means that the radius of convergence is $R = \infty$, and the interval of convergence is $(-\infty, \infty)$. □

2. Determine the power series representation for function $f(x) = \frac{1}{4+x}$, centered at $x = 0$. Determine the interval of convergence.

Solution. We know that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$. We then have

$$f(x) = \frac{1}{4+x} = \frac{1}{4\left(1 - \left(-\frac{x}{4}\right)\right)} = \frac{1}{4}\sum_{n=0}^{\infty}\left(-\frac{x}{4}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{4^{n+1}},$$

for $|x| < 1$. □