Print your name: _____

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. The problem is worth 10 points.

1. Solve the initial-value problem

$$y'' - 3y' - 4y = 0, \quad y(0) = 0, \quad y'(0) = 5.$$

Solution. We first find the general solution by first solving the characteristic equation $r^2 - 3r - 4 = 0$. It has solutions $r = \frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + 4} = \frac{3}{2} \pm \sqrt{\frac{25}{9}} = \frac{3}{2} \pm \frac{5}{2}$, meaning $r_1 = -1$ and $r_2 = 4$. Therefore the general solution is given by

$$y(x) = C_1 e^{-x} + C_2 e^{4x}.$$

We now plug in the initial conditions. First find the derivative: $y'(x) = -C_1 e^{-x} + 4C_2 e^{4x}$. Then

$$\begin{cases} y(0) = C_1 + C_2 = 0 \\ y'(0) = -C_1 + 4C_2 = 5 \end{cases}.$$

We may add the equations together to get $5C_2 = 5 \Leftrightarrow C_2 = 1$. From the first equation we then get $C_1 = -C_2 = -1$. Therefore the specific solution to the initial value problem is

$$y(x) = -e^{-x} + e^{4x}.$$

□