

Dynamics: Introductory Lectures

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Chapter I. A Discursive Introduction

1. Chaotic Dynamics: Some History
Celestial mechanics. Poincaré and sensitive dependence. The restricted 3-body problem: continuous versus discrete time. Chaotic attractors: Ueda, Lorenz, Hénon.
2. The Simplest Chaotic Systems
Two maps of the interval. Angle doubling: the squaring map on the circle. Sensitive dependence. Symbolic dynamics: the one-sided 2-shift. The solenoid.
3. Probabilistic Methods
Bernoulli and Borel: the Law of Large Numbers. Natural measures. Maxwell, Boltzmann and Jüttner: the hard sphere gas.

Chapter II. Topological Dynamics

4. Basic Concepts
Periodicity and limiting behavior. Recurrence and wandering. Transitivity and minimality. Chaos: sensitive dependence, mixing. Expansiveness.
5. Attraction and repulsion
Attracting sets. Attractors. Repelling sets and repellers.
6. Counting periodic points
Zeta functions. The Lefschetz fixed point index. How periodic orbits bifurcate. The orbit index: cascades of bifurcations. Further topological details. Problems.
7. Dimension and Entropy
Box dimension. Other dimensions. Topological entropy. A dimension inequality. Defining entropy by coverings. Piecewise monotone maps. Problems.

Chapter III. Measure Dynamics

8. Ergodic Theory: Introduction
Basic measure theory. Recurrence. Ergodicity and transitivity. Borel sets and real valued functions. Problems.
9. The Birkhoff ergodic theorem
The proof. Some consequences.
10. Measure + Topology
Ergodic theory on compact metric spaces. Existence of invariant and ergodic measures

Chapter IV. Differentiable Dynamics

11. Geodesics and Classical Mechanics
Volume preserving flows. Classical mechanics. Calculus of variations. Hamiltonian systems. Problems.
12. Liapunov Exponents and the Subadditive Ergodic Theorem
The largest Liapunov exponent. The subadditive ergodic theorem. Some linear algebra.
13. Expanding maps
Balanced measures. Circle covering maps. Absolutely continuous invariant measures

Chapter V. Degree One Circle Maps

14. The rotation number
Basic properties. Cyclic order. Irrational rotation numbers. Problems.
15. Denjoy's theorem
Distortion estimates. Denjoy counterexamples. Differential equations on the torus.
16. Dependence on Parameters.
The space of circle maps. The standard family. Stability. Analytic conjugacy and Herman rings

Appendix: Matrix estimates and the Perron-Frobenius Theorem

References

Caution: This is a preliminary version. Comments and corrections welcomed.

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