MAT 320 Pretest Monday 9/10/01

Name: **ID Number:**

The pretest consists of 5 questions on foundations and 5 questions on calculus. Answer using complete sentences and GIVE THE REASONS FOR YOUR CONCLUSIONS.

1. What are the converse and the negation of the proposition, "In the 2000 presidential election every registered Republican voted for Albert Gore."
SOLUTION Converse: In the 2000 presidential election only registered Republicans voted for Albert Gore.
Negation: In the 2000 presidential election at least one registered Republican did not vote for Albert Gore.

2. Let S be the set consisting of all nonnegative integers that are divisible by 3 and not divisible by 5. Is S empty (that is, has no elements in it), finite or infinite? If the empty set, explain why. If S is not empty, give an alternate description (formula) for the elements in S?
SOLUTION S is an infinite set. An integer $n \in S$ iff $n = 15m + r$, where $m \in \mathbb{N}_o$ and $r = 3$, 6, 9 or 12.

3. True or False? If the intersection of two sets A and B is the empty set, then either A or B must be the empty set?
SOLUTION False. Take, for example, $A = (0, 1)$ and $B = (2, 3)$.

4. True or False? If the union of two sets A and B is the empty set, then either A or B must be the empty set.
SOLUTION True. Moreover, both must be empty.

5. Let a and $b \in \mathbb{R}$. If $(a + b)^2 = a^2 + b^2$, what can you conclude about a and b?
SOLUTION

$$a^2 + 2ab + b^2 = (a + b)^2 = a^2 + b^2$$

iff $2ab = 0$ iff either a or $b = 0$.

6. True or False?
Let a, b and c be arbitrary real numbers.
(a) There always exists an $x \in \mathbb{R}$ such that $x^2 + ax + b = 0$.
SOLUTION False. Take, for example, $a = 0$ and $b = 1$.

(b) There always exists an $x \in \mathbb{R}$ such that $x^3 + ax^2 + bx + c = 0$.
SOLUTION True. Since $\lim_{x \to \pm\infty} x^3 + ax^2 + bx + c = \pm\infty$, the polynomial is positive for some $X > 0$ and negative for some $Y < 0$. Hence it must vanish someplace in (Y, X).

7. Define two functions $f(x) = e^{\ln(x)}$ and $g(x) = \ln(e^x)$.
(a) What are the natural domains of the definition for the functions?
SOLUTION The natural domain of definition for f is $(0, \infty)$; and for g, $(-\infty, \infty)$.

(b) Justify the formula $\frac{d}{dx} e^{\ln(x)} = 1 = \frac{d}{dx} \ln(e^x)$.
SOLUTION

$$f(x) = x \text{ for } x \in (0, \infty) \text{ and } g(x) = x \text{ for } x \in (-\infty, \infty).$$

8. Compute $\frac{d}{d\theta} \cos^2(\theta)$.
SOLUTION

$$\frac{d}{d\theta} \cos^2(\theta) = -2\cos(\theta)\sin(\theta) = -\sin(2\theta).$$

9. Evaluate $\int \cos^2(\theta) d\theta$.
SOLUTION

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1.$$

Hence

$$\int \cos^2(\theta) d\theta = \int \frac{\cos(2\theta) + 1}{2} d\theta = \frac{\sin(2\theta)}{4} + \frac{\theta}{2} + C.$$

10. Describe how you would compute (without using a calculator) $\ln(3)$ to three decimal places.
SOLUTION Use the formula $\ln(3) = \int_1^3 \frac{1}{x} dx$. Subdivide the interval $(1, 3)$ into n equal subintervals. Use the subdivision to inscribe a union of rectangles R_0 inside the region $R = \{(x, y) \in \mathbb{R}^2;\ 1 < x < 3,\ 0 < y < \frac{1}{x}\}$. Use the subdivision to circumscribe a union of trapezoids R_1 about the region R. Then

$$\text{Area } R_0 < \text{ Area } R < \text{ Area } R_1.$$

Now

$$\ln(3) = \text{ Area } R$$

and by choosing n sufficiently large

$$\text{Area } R_1 - \text{ Area } R_0 < .0005.$$