MAT 320 Quiz #2 with SOLUTIONS Friday 10/19/01

Name: **ID Number:**

Determine whether each of the following 5 series $\sum_{n=1}^{\infty} a_n$ converges or diverges. If the series converges, determine if convergence is absolute or conditional. Explain what tests you are using to reach your conclusion.

1. $a_n = (-1)^n \frac{1}{\sqrt{n}}$.
Solution Since $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ and $\frac{1}{\sqrt{n}}$ is monotonic decreasing, the Cauchy test gives *convergence*. The series converges *conditionally* by the integral test.

2. $a_n = \frac{\sin n}{n^2}$.
Solution Since $\left|\frac{\sin n}{n^2}\right| \leq \frac{1}{n^2}$, the series *converges absolutely* by the comparison test.

3. $a_n = \frac{2^n}{n!}$.
Solution Since $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2}{n+1} = 0$, the series *converges absolutely* by the ratio test.

4. $a_n = \left(\frac{n^2}{2n^2+3n+4}\right)^{\frac{n}{2}}$.
Solution Since $\lim_{n\to\infty} \sqrt{\frac{n^2}{2n^2+3n+4}} = \frac{1}{\sqrt{2}} < 1$, the series *converges absolutely* by the root test.

5. $a_n = (-1)^{3n} \frac{n^2}{n^4+3n^2+5}$.
Solution The series *converges absolutely* by the asymptotic comparison test (with $\sum_{n=1}^{\infty} \frac{1}{n^2}$).