MAT 320 Quiz #1 Wednesday 10/10/01

Name: **ID Number:**

Each of the following 5 assertions is FALSE. Describe a counterexample to each.

1. Every bounded sequence $\{a_n\}$ converges.
COUNTEREXAMPLE: $\{1, -1, 1, -1, ...\}$ or written differently $a_n = (-1)^n$.
2. The intersection of an infinite sequence of proper open intervals $I_n = (a_n, b_n)$ (thus $a_n \leq a_{n+1} < b_{n+1} \leq b_n$ for all positive integers n) is nonempty.
COUNTEREXAMPLE: Let $a_n = 0$ and $b_n = \frac{1}{n+1}$. (Every positive r is not in $\left(0, \frac{1}{n+1}\right)$ for some (hence many) n.)
3. A convergent series $\{S_n = \sum_{i=0}^{n} a_i\}$ converges absolutely.
COUNTEREXAMPLE: $a_n = (-1)^n \frac{1}{n}$.
4. If the sequence $\{a_n\}$ is not bounded from above, then $\lim_{n\to\infty} a_n = \infty$.
COUNTEREXAMPLE:$\{1, 1, 1, 2, 1, 3...\}$ or written differently $a_n = \begin{cases} 1 & \text{for odd } n \\ \frac{n}{2} & \text{for even } n \end{cases}$.
5. Let $\{a_n\}$ be a sequence and form the series $\{S_n\}$ of partial sums (thus $S_n = \sum_{i=0}^{n} a_i$). If $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} S_n$ exists.
COUNTEREXAMPLE: $a_n = \frac{1}{n}$.