# MAT 319 Quiz #2 with solutions Friday 2/14/03

Determine whether each of the following 2 assertions is TRUE or FALSE. Give a brief explanation for your answer.

**1.** If $f$ and $g$ are functions defined on $(-1, 1)$ and $\lim_{x\to 0} f(x)g(x) = 3$, then $\lim_{x\to 0} f(x)$ exists. **SOLUTION:** FALSE. Let $f(x) = \begin{cases} \frac{1}{x} \text{ for } x \neq 0 \\ 0 \text{ for } x = 0 \end{cases}$ and $g(x) = 3x$.

**2.** If $f$ and $g$ are functions defined on $(-1, 1)$ and $\lim_{x\to 0} g(x) = 0$, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ can not exist. **SOLUTION:** FALSE. Let $f(x) = x = g(x)$.

**3.** Define carefully what $\lim_{x\to a} f(x) = L$ means. **SOLUTION:** For all $\epsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \epsilon$ for $0 < |x - a| < \delta$.

**4.** State the Heine-Borel theorem. **SOLUTION:** Every cover of a bounded closed interval by open intervals has a finite subcover.

**5.** For the positive integer $n$, let $I_n = (0, \frac{1}{n})$. What real numbers belong to the intersection $\cap_{n=1}^{\infty} I_n$? Prove that your answer is correct. **SOLUTION:** $\cap_{n=1}^{\infty}(0, \frac{1}{n}) = \emptyset$. For if $\epsilon > 0$, there exists a positive integer $N$ such that $\frac{1}{N} < \epsilon$. Thus $\epsilon \notin I_n$ for any (some is all that we need) $n \geq N$.