

MAT312-AMS351, FALL 02

Applied Algebra

HOMEWORK SET 1

Due Thursday, September 12, 2002

1. From section 1.1 of textbook do problems 1, 6.
2. From section 1.2 do problems 1, 2, 6.
3. (a) Let $m, n \in \mathbb{Z}$. Prove the identity:

$$\sum_{i=0}^k \binom{m}{i} \binom{n}{k-i} = \binom{m+n}{k}.$$

Hint: Consider the polynomial equation

$$\sum_{k=0}^{m+n} \binom{m+n}{k} z^k = (1+z)^{m+n} = (1+z)^m (1+z)^n.$$

- (b) Show that if $n \geq 1$, then

$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}.$$

4. Show that if $n \in \mathbb{Z}_{>0}$, then

$$\sum_{k=0}^n (-1)^k \binom{n}{k} = 0.$$

5. Do worksheet # 1. If at all possible, consult the Maple program that accompanies this worksheet only after you have completed it.
6. From section 1.1 do problem 2. This gives a different way to determine the function $p(n)$ of worksheet # 1.