MAT312/AMS351 Applied Algebra – Fall 2002

Quiz #7 with solutions

12/3/2002

Problems 1 & 2: True or false: (Circle the correct answers.) Let G be a group and a, b two distinct elements in G with neither the identity.

T F (1) There exists an integer r such that $a^r = b$.

T F (2) $(ab)^{-1} = b^{-1}a^{-1}$.

SOLUTION: (1) is False. Take $G = S(3)$, $a = (1, 2)$ and $b = (1, 2, 3)$.
(2) is True.

The next three problems concern the group $G = S(4)$ and its cyclic subgroup $H =< (1, 2, 4, 3) >$.

Problem 3: How many distinct left H cosets are there in G?

SOLUTION: The number of distinct left H cosets in $G = \frac{o(G)}{o(H)} = \frac{24}{4} = 6$.

Problem 4: What are the orders of two groups G and H?

SOLUTION: $o(G) = 4! = 24$ and $o(H) = o((1, 2, 4, 3)) = 4$.

Problem 5: Without actually computing any orders, what theorem allows one to conclude what are the possibilities for the orders of elements of G? What are the possible orders of the permutations in G? Do all of these actually occur?

SOLUTION: Lagrange's theorem tells us that the orders of elements of G must divide the order of $G = 24$. Thus the possible orders of elements of $S(4)$ are 1, 2, 3, 4, 6, 8, 12 and 24. The last of these certainly does not occur since $S(4)$ is not cyclic.