MAT312/AMS351 Applied Algebra – Fall 2002

Quiz #6 with solutions

11/26/2002

Name:
SB ID:

Problems 1 & 2: True or false: (Circle the correct answers.)

T F (1) There exists a positive integer r such that $3^r \equiv 1 \mod 6$.

T F (2) All groups containing 6 elements are abelian.

SOLUTION: (1) is False because $[3]_6$ is a zero divisor.
(2) is False because $S(3)$ is not commutative and $|S(3)| = 6$.

Problem 3: Let G be a group and g, one of its members. Define the order of g.

SOLUTION: The element g has *finite* order if there exists a positive integer n such that $g^n = e$; the smallest such n is then the *order* of g. If g does not have finite order, then its order is said to be *infinite*.

Problem 4: What are all the proper subgroups of $\mathbb{Z}_6$?

SOLUTION: The proper subgroups of $\mathbb{Z}_6$ contain 1, 2 or 3 elements. The subgroup containing one element is $\{[0]_6\}$. The subgroup containing two elements is $\{[0]_6, [3]_6\}$. The subgroup containing three elements is $\{[0]_6, [2]_6, [4]_6\}$. These are all the proper subgroups of $\mathbb{Z}_6$.

Problem 5: Label the four vertices of the square in the counterclockwise order as 1, 2, 3 and 4. What permutation in $S(4)$ corresponds to reflection the the diagonal joining vertex 1 to vertex 3?

SOLUTION: The transposition $(2, 4)$ corresponds to this reflection.