

MAT312/AMS351 Applied Algebra – Fall 2002

Quiz #5 with solutions.

11/5/2002

**Name:**

**SB ID:**

**Problems 1 & 2:** True or false: (Circle the correct answers.) Let  $n$  be an integer  $\geq 2$ .

T F (1) If  $\pi$  and  $\tau \in S(n)$  are odd permutations, then so is  $\pi\tau$ .

T F (2) For all  $\tau \in S(n)$ , both  $\tau$  and  $\tau^2$  have the same fixed points.

SOLUTION: (1) is false, since  $\text{sgn}(\pi\tau) = \text{sgn}(\pi)\text{sgn}(\tau)$ .

(2) is false for every transposition.

**Problem 3:** Give an example of a permutation  $\pi \in S(10)$  that is not a transposition and has order 2.

SOLUTION:  $(1, 2)(3, 4)$ .

**Problem 4:** Define what it means for a group  $(G, *)$  to be abelian.

SOLUTION: For all  $x$  and  $y \in G$ ,  $x * y = y * x$ .

**Problem 5:** Give an example of a group  $(G, *)$  with precisely 3 elements. Is this group abelian?

SOLUTION: The abelian group  $(\mathbb{Z}_3, +)$  is such an example.