MAT312/AMS351 Applied Algebra – Fall 2002

Quiz #3 WITH SOLUTIONS

10/17/2002

Name:
SB ID:

Problems 1 & 2: True or false: (Circle the correct answers.) Let a, b, c and d be positive integers.

T F (1) For every positive integer n, the congruence classes $\mathbb{Z}_n$ always contain nonzero zero divisors.

T F (2) Every nonempty set of positive integers contains a largest element.

SOLUTION: (1) is FALSE for primes n.
(2) is FALSE for $\{1, 2, 3, 4, ..., \}$.

Problem 3: Let n be an integer ≥ 2. Define what it means for the nonzero congruence class $[a]_n \in \mathbb{Z}_n$ to be a zero divisor.

SOLUTION: There exists a $b \in \mathbb{Z}$ such that

$$b \not\equiv 0 \mod n$$

and

$$ab \equiv 0 \mod n.$$

Problem 4: Determine all $x \in \mathbb{Z}$ that solve the linear congruence

$$6x \equiv 9 \mod 15.$$

SOLUTION: Since $(6, 15) = 3|9$, an equivalent equation is

$$2x \equiv 3 \mod 5.$$

Since $[2]_5^{-1} = [3]_5$, the solution is given as

$$x = [9]_5 = [4]_5.$$

Problem 5: Let p be an odd prime, prove that $\varphi(2p) = p - 1$.

SOLUTION: $\varphi(2p) = \varphi(2)\varphi(p) = 1(p - 1)$.