MAT312/AMS351 Applied Algebra – Fall 2002

Quiz #2

10/1/2002

Name:
SB ID:

Problems 1 & 2: True or false: (Circle the correct answers.) Let a, b, c and d be positive integers.

T F (1) If there exist integers r and s such that $ra + sb = d$, then $d = \gcd(a, b)$.
T F (2) $\text{lcm}(a, b) > \gcd(a, b)$.

SOLUTION. (1) is FALSE. Counterexample: $2 \cdot 1 + 2 \cdot 1 = 2$, but $(1, 1) = 1$.
(2) is FALSE: If $a = b$, then $\text{lcm}(a, b) = \gcd(a, b)$.

Problem 3: Express 24 and 102 as products of primes and use this information to calculate $\gcd(12, 102)$ and $\text{lcm}(12, 102)$.

SOLUTION. $24 = 2^3 3$ and $102 = 2 \cdot 3 \cdot 17$. Therefore $\gcd(12, 102) = 2 \cdot 3 = 6$ and $\text{lcm}(12, 102) = 2^2 3 \cdot 17 = 204$.

Problem 4: Use the Euclidean algorithm to calculate the gcd of -24 and -102.

SOLUTION.

$$-102 = 5(-24) + 18,$$
$$-24 = (-2)18 + 12,$$
$$18 = 1 \cdot 12 + 6$$

and

$$12 = 2 \cdot 6.$$

So we conclude that $(-24, -102) = 6$, as expected.

Problem 5: How many elements does $G_8 = \mathbb{Z}_8^*$ contain? List them.

SOLUTION. G_8 has 4 elements, they are $[1]_8, [3]_8, [5]_8$ and $[7]_8$.