MAT312/AMS351 Applied Algebra – Fall 2002

Quiz #1

9/19/2002

Name:
SB ID:

Problems 1 & 2: True or false: (Circle the correct answers.) Let a, b, c and d be positive integers.

T F (1) If $a|c$ and $b|c$, then $ab|c$.

T F (2) If $(a,b) = 1$ and $(c,d) = 1$, then $(ac, bd) = 1$.

SOLUTION (1) is False. Counterexample: $4|12$ and $6|12$; but $24 \not| 12$.

(2) is False. Counterexample: $(3,4) = 1$ and $(4,5) = 1$, but $(12,20) = 4$.

Problem 3: State the well ordering principle.

SOLUTION A nonempty set of integers that is bounded from below contains a least element.

Problem 4: Define (a, b), the greatest common divisor of the two positive integers a and b.

SOLUTION The gcd of a and b is the unique positive integer d that divides both a and b and has the additional property that whenever an integer c divides both a and b, it also divides d.

Problem 5: Find $[3]_7^{-1}$.

SOLUTION By inspection,

$$[3]_7[2]_7 = [6]_7,$$
$$[3]_7[3]_7 = [2]_7,$$
$$[3]_7[4]_7 = [5]_7,$$

and

$$[3]_7[5]_7 = [1]_7.$$

Thus $[3]_7^{-1} = [5]_7$.