Economics 362 – Practice Problems

1. Suppose that you are at date 0, the current stock price is \$100 and the evolution of the price in the two following periods can be represented by the following tree



Compute the value at date 0 of an European put option with exercise price \$100 expiring at T = 2, assuming that the interest rate in each period is 5%. Then do the same for an American put option with the same exercise price and expiration date.

Answer. We first compute the risk neutral probabilities. At the node at which S = 110 we have

$$p_u^1 = \frac{(1.05) \times 110 - 100}{121 - 100} = 0.73810$$

and at the node at which S = 90 we have

$$p_d^1 = \frac{(1.05) \times 90 - 81}{100 - 81} = 0.71053.$$

Finally, at the initial node we have

$$p^{0} = \frac{(1.05) \times 100 - 90}{110 - 90} = 0.75.$$

Using this risk neutral probabilities, we can compute the prices of the European put option.

$$c_u^1 = 0$$

$$c_d^1 = \frac{1}{1.05} \left(0.71053 \times 0 + (1 - 0.71053) \times 19 \right) = 5.238$$

$$c^0 = \frac{1}{1.05} \left(0.75 \times 0 + 0.25 \times 5.4999 \right) = 1.3095$$

For an American put option the only change is for c_d^1 . In this case immediate exercise is optimal, so that $c_d^1 = 10$. Thus, at time zero the price is

$$c^{0} = \frac{1}{1.05} \left(0.75 \times 0 + 0.25 \times 10 \right) = 2.3810$$

- 2. Buffelhead's stock price is \$220 and could halve or double in each 6month period. A 1-year call option on Buffelhead has an exercise price of \$165. The interest rate is 10% per semester.
 - (a) What is the value of the Buffelhead call?
 - (b) Suppose that in month 6 the Buffelhead stock price is \$110. How at that point could you replicate an investment in the stock by a combination of call options and risk-free lending? Show that your strategy does indeed produce the same returns as those from an investment in the stock.
 - (c) Consider now an American put option on Buffelhead stock with an exercise price of \$220. Would you ever want to exercise the put early?
 - (d) Calculate the value of the American put and the value of an equivalent European put option.

Answer. We first compute the risk neutral probabilities. In this case The risk neutral probability is

$$p = \frac{1.1 - 0.5}{2 - 0.5} = 0.4$$

(a) The value of the call option price is

$$c = \frac{1}{(1.1)^2} \left((0.4)^2 \times (4 \times 220 - 165) + 2(0.4)(0.6) \times (220 - 165) \right) = 116.36$$

(b) If the stock goes up its price will be 220 an the value of the option will be (220 - 165); if it goes down, its price will be 55 and the value of the option zero. Let X be the amount of money paid by the bond at maturity and n the number of call options. We want

$$X + n \times (220 - 165) = 220$$

 $X + n \times 0 = 55$

The solution is: [X = 55, n = 3].

(c) First compute the prices at of the European put option

$$c_u^1 = 0$$

$$c_d^1 = \frac{1}{1.1} \left(0.4 \times 0 + 0.6 \times (220 - 55) \right) = 90$$

$$c^0 = \frac{1}{1.1} \left(0.4 \times 0 + 0.6 \times 90 \right) = 49.091$$

The only point at which the option can be exercised early is when price is 110. In that case $c_d^1 = 220 - 110 = 110$, which is better than 90. Therefore at that point early exercise is optimal.

(d) The value of the European put option has been computed at the previous point. For the American option, the price at time 0 is

$$c^{0} = \frac{1}{1.1} \left(0.4 \times 0 + 0.6 \times 100 \right) = 54.545.$$