Economics 362 – Practice 3

- 1. Suppose that the risk free interest rate is $r_f = 4\%$ and the expected return on the market portfolio is $\overline{r}_M = 7\%$. The variance of the market portfolio is $\sigma_M^2 = .25$
 - (a) Using CAPM, find the expected rate of return of a security having a $\beta = 1.5$.
 - (b) Suppose that a security has an expected return of 5%. Use CAPM to determine the β of the security and its covariance with the market portfolio.

Answer.

(a) According to CAPM the expected return should be

$$E(r_i) = r_f + \beta \left(\overline{r}_M - r_f\right).$$

In our case $r_f = 4\%$, $\overline{r}_M = 7\%$ and $\beta = 1.5$. Therefore

$$r_i = 4 + 1.5 \left(7 - 4\right) = 8.5\%.$$

(b) If $r_i = 5$ then

$$5 = 4 + \beta (7 - 4)$$

and solving for β we obtain $\beta = \frac{1}{3}$. To find the covariance remember that $\beta = \frac{Cov(r_i, r_M)}{\sigma_M^2}$, so that

$$Cov\left(r_{i}, r_{M}\right) = \beta \sigma_{M}^{2}.$$

Therefore $Cov(r_i, r_M) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.

2. Here are returns and standard deviations for four investments.

	Return	Standard Deviation
Treasury bills	0.06	0
Stock P	0.10	0.14
Stock Q	0.145	0.28
Stock R	0.21	0.26

Calculate the standard deviations of the following portfolios

- (a) 50% in treasury bills, 50% in stock P.
- (b) 50% each in Q and R, assuming the shares have
 - i. perfect positive correlation
 - ii. perfect negative correlation
 - iii. no correlation

Answer.

(a) The general formula for a portfolio of two securities is

$$Var\left(\alpha r_a + (1-\alpha)r_b\right) = \alpha^2 \sigma_a^2 + (1-\alpha)^2 \sigma_b^2 + 2\alpha \left(1-\alpha\right) \rho_{ab} \sigma_a \sigma_b.$$

If $\alpha = 0.5$ and security *a* is a treasury bill, $\sigma_a = 0$ and $\rho_{ab} = 0$. If security *b* is stock *P* then $\sigma_b = 0.14$. We conclude

$$Var \left(\alpha r_a + (1 - \alpha) r_b\right) = (0.5)^2 (0.14)^2$$

and the standard deviation is

$$\sigma = \sqrt{\left(0.5\right)^2 \left(0.14\right)^2} = 0.07.$$

- (b) If $\alpha = 0.5$, security *a* is stock Q, and security *b* is stock R then $\sigma_a = 0.28$, $\sigma_b = 0.26$.
 - i. If Q and R have perfect positive correlation then $\rho_{ab}=1$ then

$$Var (\alpha r_a + (1 - \alpha) r_b)$$

= (0.5)² (0.28)² + (0.5)² (0.26)² + 2 (0.5)² × 1 × 0.28 × 0.26
= 0.0729

and the standard deviation is

$$\sigma = \sqrt{0.0729} = 0.27$$

ii. If Q and R have perfect negative correlation then $\rho_{ab}=-1$ then

$$Var\left(\frac{1}{2}r_{a} + \frac{1}{2}r_{b}\right)$$

= $(0.5)^{2}(0.28)^{2} + (0.5)^{2}(0.26)^{2} - 2(0.5)^{2} \times 1 \times 0.28 \times 0.26$
= 0.0001

and the standard deviation is

$$\sigma = \sqrt{0.0001} = 0.01.$$

iii. If Q and R have zero correlation then $\rho_{ab}=0$ then

$$Var\left(\frac{1}{2}r_a + \frac{1}{2}r_b\right) = (0.5)^2 (0.28)^2 + (0.5)^2 (0.26)^2 = 0.0365$$

and the standard deviation is

$$\sigma = \sqrt{0.0365} = 0.19105$$

- 3. True or false?
 - (a) The CAPM implies that if you could find an investment with a negative beta, its expected return would be less than the risk-free interest rate.
 - (b) The expected return of an investment with a beta of 2 is twice as high as the expected return on the market.
 - (c) If a stock lies below the securities market line, it is undervalued.

Answer.

(a) True. According to CAPM the expected return $E(r_i)$ of a security with beta β_i is

$$E(r_i) = r_f + \beta_i \left(\overline{r}_M - r_f\right).$$

Therefore, if $\overline{r}_M - r_f > 0$ (as it is normally the case), the expected return of a security with a negative β_i should be less than r_f .

(b) False. In this case the expected return is

$$E(r_i) = r_f + 2(\overline{r}_M - r_f) = 2\overline{r}_M - r_f$$

which is less than $2\overline{r}_M$ whenever $r_f > 0$.

(c) False. The fair price should be $\widehat{P}_t = \frac{E(P_{t+1})}{1+r_f+\beta_i(\overline{r}_M-r_f)}$. If a stock lies below the security market line then $E(r_i) < r_f + \beta_i (\overline{r}_M - r_f)$, so the actual price is

$$P_t = \frac{E\left(P_{t+1}\right)}{1 + E\left(r_i\right)} > \widehat{P}_t.$$

Therefore the security is overpriced.

4. True or false?

- (a) Investors demand higher expected rates of reurn on stocks with more variable rates of return.
- (b) The CAPM predicts that a security with $\beta = 0$ will offer a zero expected return.
- (c) An investor who puts \$10,000 in Treasury bills and \$20,000 in the market portfolio will have a beta of 2.
- (d) Investors demand higher expected rates of returns from stocks with returns that are very sensitive to fluctuations in the stock market.

Answer.

- (a) False. Investors demand higher expected rates of return on stocks with returns more correlated to the return on the market portfolio (or other macroeconomic risk factors).
- (b) False. A security with a beta of zero will offer the risk-free rate of return.
- (c) False the beta will be

$$\beta = \frac{1}{3} \times 0 + \frac{2}{3} \times 1 = \frac{2}{3}.$$

(d) True.