The Wald Tests for Testing Hypotheses

Eco321: Econometrics

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An Example: The Determinants of Wage

The Econometric Model

$$\log(Wage) = \beta_0 + \beta_1 \text{Age} + \beta_2 \text{Education} + \beta_3 \text{Female} + \epsilon$$

Matlab Commands

The dependent and explanatory variables

```
% open dataset
data = csvread('wages2.csv') ;

% Working People
data = data (data(:,10)>0, :);

% Depedent Variable: Wage
y = data(:,10) ; LogWage = log(y) ;

% Explanatory Variables
Age = data(:,3) ; Education = data(:,5) ; Female = data(:,6) ;
X = [Age Education Female];
```

Matlab Commands

Estimation by OLS

```
% Regress LogWage on X
Out = regstats (LogWage, X, 'linear') ;

% betahat and varaince-covariance matrix
betahat = Out.beta ; Var = Out.covb ;
```

The Wald Test

Testing Hypothesis

$$H_0 : R\beta = r$$

where R is a $q \times k$ matrix

The Test Statistic

$$W = (R\hat{\beta} - r)'[RVar(\hat{\beta})R']^{-1}(R\hat{\beta} - r) \quad \overset{a}{\sim} \quad \chi^2_q$$

The Wald Test

Example 1

$$H_0 : \beta_2 = 0$$

$$\underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}}_{\beta} = \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{r}$$

The Wald Test

Matlab Commands

```
% R and r matrices
R = [0 0 1 0] ; r = 0 ;

% The Wald test statistic
diff = R*betahat - r ; Vdiff = R*Var*R' ;
W=diff'*inv(Vdiff)*diff ;

% The rejection region
df = size(R,1); alpha = 0.05 ;
Reject = chi2inv(1-alpha, df);
```

The Wald Test

Example 2

$$H_0 : \beta_2 = \beta_3$$

$$\begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

The Wald Test

Example 3

$$H_0 : \beta_1 = 0 \quad \text{and} \quad \beta_2 = 0$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The Wald Test

Test for structural change

For Men:

$$\log(Wage_M) = X_M\beta_M + \epsilon_M$$

For Women:

$$\log(Wage_W) = X_W\beta_W + \epsilon_W$$

Testing Hypothesis

$$H_0 : \beta_M = \beta_W$$

where β_M and β_W are $k \times 1$ matrices

The Wald Test

- Divide the whole sample into men's and women's samples.
- Run Regressions, separately.
- Find $\hat{\beta_M}$, $Var(\hat{\beta_M})$ and $\hat{\beta_W}$, $Var(\hat{\beta_W})$

The Test Statistic

$$W = (\hat{\beta_M} - \hat{\beta_W})'[Var(\hat{\beta_M}) + Var(\hat{\beta_W})]^{-1}(\hat{\beta_M} - \hat{\beta_W}) \overset{a}{\sim} \chi^2_k$$

See the Professor's class handout, "Comparing Women's and Men's Wage Regressions", on Blackboard.